

## Solutions to Attendance Quiz for Lecture 0

1. Find the values of the numbers  $x$ ,  $y$ , and  $z$  if they satisfy the following system of linear equations:

$$x + y + z = 6 \quad , \quad x + 2y + 3z = 14 \quad , \quad x + 4y + 9z = 36 \quad .$$

**Sol. of 1:** We first **eliminate** the variable  $x$ , expressing it in terms of  $y$  and  $z$ . Using the **first equation**

$$x + y + z = 6 \quad ,$$

and pretending that we already know the values of  $y$  and  $z$ , we get

$$x = 6 - y - z \quad .$$

So if and when we would know what the values of  $y$  and  $z$  are, we would also know the value of  $x$  (using **back substitution**).

We are left with two equations

$$x + 2y + 3z = 14 \quad , \quad x + 4y + 9z = 36 \quad .$$

Using the fact that  $x = 6 - y - z$ , we **plug it in**, getting rid of any mention of  $x$ .

$$(6 - y - z) + 2y + 3z = 14 \quad , \quad (6 - y - z) + 4y + 9z = 36 \quad .$$

**Simplifying:**

$$y + 2z = 8 \quad , \quad 3y + 8z = 30 \quad .$$

We are now left with the much easier task of solving a system of **two** equations with two unknowns. Let's do it.

From the first equation

$$y = 8 - 2z \quad .$$

So if and when we will know the value of  $z$ , we would know the value of  $y$  as well (and hence also  $x$ ).

Looking at the second equation

$$3y + 8z = 30 \quad ,$$

and **substituting**  $y = 8 - 2z$  into it, we get

$$3(8 - 2z) + 8z = 30 \quad .$$

Opening up parantheses

$$24 - 6z + 8z = 30 \quad ,$$

and simplifying, we get

$$2z = 6 \quad .$$

This is a “system” of **one** equation with one unknown. Going back to second grade, we solve it and get

$$z = \frac{6}{2} = 3 \quad .$$

We now do the **back journey**. From  $y = 8 - 2z$  we get

$$y = 8 - 2 \cdot 3 = 2 \quad ,$$

and from  $x = 6 - y - z$ , we get

$$x = 6 - 2 - 3 = 1 \quad .$$

**Ans. to 1:**  $x = 1$ ,  $y = 2$ ,  $z = 3$ .

It is always good to check! Indeed

$$1 + 2 + 3 = 6 \quad , \quad 1 + 2 \cdot 2 + 3 \cdot 3 = 14 \quad , \quad 1 + 4 \cdot 2 + 9 \cdot 3 = 36 \quad ,$$

and we did not mess up!

**Comment:** About 80 percent of the people got it perfectly. Most other people did it the right way, but made one or more computational error. That’s why it is good to check! In a quiz or exam, if you do it the right way, but get the wrong answer, and realize that you are wrong by checking your answer, you would get generous partial credit (up to 40 percent of the full points). But if you do not check, and do not realize that you messed up, you would get no partial credit.