Solutions to Assigned True-False Questions in "Elementary Linear Algebra", 2e, by L.E. Spence et. al.

By Dr. Z.

IMPORTANT: PLEASE first attempt to do the problems by yourself. This is for checking purposes only. When I check the homework, I have no way of knowing whether you just copied this, but you would be cheating yourself, since the point is to **understand** the concepts.

Disclaimer: Not responsible for any errors. The first person to detect an error will get one dollar.

Section 1.1 (p. 12)

37. T (p. 5, line 10);

38 T (Theorem 1.2(a), p.9)

39 T (according to the book, but False according to me, any vector may be viewed as a matrix, but a vector is a one-dimensional array, and a matrix is two-dimensional).

40 F (it is the zero **matrix**, a matrix stays a matrix!)

41 F (only true for square-matrices, the transpose of an $m \times n$ matrix is an $n \times m$ matrix)

42 T (according to the book)

43 F (its rows are 3×1 vectors)

44 F (lies in row 3 and and column 4)

45 T

46 F (mn entries **not** $m + n$)

47 T

48 T (since $-B = (-1)B$)

49 T (by definition of transpose)

50 F (A and B have different sizes, hence can't be equal)

51 T (since the (i, j) entry of $3A$ is $3a_{i,j}$ adding them up gives the same thing as adding up $a_{i.j}$ and multiplying by 3)

52 T (since adding numbers is commutative, and adding matrices is done entry-by-entry)

53 T (ditto)

54 T (By combining parts (a) and (b) of Theorem 1.2 , p. 7)

55 T

56 T (If A is $m \times n$ then A^T is $n \times m$, so in order for $A + A^T$ to be defined m be the same as n.)

Section 1.2 (pp. 25-26)

45 T (by definition)

46 F (e.g. $[1, -1]^T = 1 \cdot [1, 0]^T + (-1 \cdot [0, 1]^T)$, and that's the only way to express $[1, -1]^T$ as a linear combination of $[1, 0]^T$ and $[0, 1]^T$.)

47 T ($[a, b]^T = a\mathbf{e_1} + b\mathbf{e_2}$)

48 T (bottom of p. 17)

49 T (make all the coefficients 0)

50 F (it is a 2×1 vector)

51 F (rows $\rightarrow$ columns)

52 F (The product of a matrix A with the standard vector e_i is its i-th column)

53 T (see def. of A_θ on p. 23, take $\theta = \pi$)

54 F (it should be R^m)

55 F (parallel $\rightarrow$ non-parallel)

56 T (p. 17 towards the bottom)

57 F (the only non-zero component must be 1 in order to qualify as a standard vector)

58 T (since $cA = Ac$)

59 F (The obvious counter-example is when A is the zero matrix, but whenever the rank is less than n there are infinitely many such non-zero $\mathbf{u}$.)

60 T (except they should have said **counter-clockwise**)

61 F (clockwise $\rightarrow$ counter-clockwise)

62 F (for the same reason as #59, with $\mathbf{u} - \mathbf{v}$ instead of $\mathbf{u}$)

63 T (By definition on p. 19)

Section 1.3 (pp. 39-40)

57 F (It may have no solutions).

58 F (If it has more than one solution, then it has infinitely many solutions).

59 T (All elementary row operations are reversible).

60 F ("row echelon" $\to$ "reduced row echelon form") .

61 T (See 60) .

62 T (This is the whole purpose of elementary row operations) .

63 F (There is a unique matrix in **reduced** row echelon form) .

64 T (See 63) .

65 T (That's the whole point) .

66 F (It is inconsistent if it has a row all zeros except the last one, but it is very possible that an inconsistent system, in addition to having such a row also has rows with all zeros).

67 T (If you have such a scenario, in everyday language it means $0 \cdot x_1 + \ldots + 0 \cdot x_n + NotZero = 0$, and this is **nonsense**).

68 T (By definition) .

69 F ($n \times n \to m \times n$) .

70 T (The rightmost column corresponds to the right hand side) .

71 T (By definition of basic variable) .

72 T (That's the whole point of the Gaussian Elimination algorithm) .

73 F ("scalar" $\to$ "**non-zero** scalar") .

74 T (That's the whole point of the general solution, that you can plug-in specific values to the free variables and get yet-another-solution) .

75 F (E.g. $\{x_1 + x_2 + x_3 + x_4 = 1, x_1 + x_2 + x_3 + x_4 = 2\}$ has two equations and four variables, and **no solution**. However this is true if the system is consistent).

76 T (By definition of what it means to be a solution).

Section 1.4 (p. 54)

53 T (By definition, it is also true for row echelon form) .

54 F (There is some freedom of choice, but the final outcome is always the same) .

55 T (The second pass (from bottom to top) is in making it into reduced row echelon form) .

56 T (But it is not forbidden) .

57 T (By definition) .

58 T (See(b) in box at bottom of p. 48).

59 F (Rank plus nullity equals n (the number of columns), $3 + 2 \neq 8$) .

60 F (E.g. $\{x_1 + x_2 = 1, 2x_1 + 2x_2 = 2\}$ is equivalent to $\{x_1 + x_2 = 1\}$ the former has two equations the latter has one equation).

61 T (The number of equations may change, but the number of variables stays the same).

62 T (by definition of **matrix-vector product** in p. 19)

63 T (**b** is **not** a linear combination of the columns of A)

64 F (E.g. $\{x_1 = 1, x_2 = 1, x_1 + x_2 = 2\}$ has reduced row echelon form with the last row being all 0, but only has one solution).

65 F (It may have such rows and still be inconsistent).

66 T (everything below the pivot entry must be 0 (since it is row-echelon form), and everything above it (since it is reduced row echelon form), and the entry itself must be 1 (since it is reduced row echelon form), so such pivot column has all zeros except at one place (the pivot entry) where it is 1).

67 T .

68 F ('number of rows' $\rightarrow$ 'number of columns').

69 T (by definition of 'linear combination').

70 T (Every non-zero row has a pivot, so the first pivot position is in row 1, the second in row 2, etc.)

71 F ('column 3' $\rightarrow$ 'row 3').

72 T (See box at the middle of p. 48).

Section 1.6 (p. 73)

45 T (By definition of Span, p. 66) .

46 T (The span of $\mathbf{0}$ is the set $\{c\mathbf{0}; -\infty < c < \infty\}$. But $c\mathbf{0} = \mathbf{0}$ for all c, so it only contains one vector, the boring zero vector $\{\mathbf{0}\}$.

47 T (Th. 1.6 (b)).

48 F ('the rank of A is n' $\rightarrow$ 'the rank of A is m')

49 T (Th. 1.6(a) with m replaced by n)

50 T (Given a set of vectors $\{\mathbf{u_1}, \ldots \mathbf{u_k}\}$, every vector, for example, $\mathbf{u_1}$, can be written as $1 \cdot \mathbf{u_1} + 0 \cdot \mathbf{u_2} + \ldots + 0 \cdot \mathbf{u_k}$, hence is a linear combination).

51 T (Every linear combination of members of $\mathcal{S}_1$ is also a linear combination of members of $\mathcal{S}_2$)

52 F (Simple example, $Span(\{[1,0]^T\}$ and $Span(\{[2,0]^T\}$ are equal).

53 F (Simple example, you can always add $\mathbf{0}$ without changing the Span).

54 F ('$\mathbf{v}$ is in S' $\rightarrow$ '$\mathbf{v}$ is in the **Span** of S').

55 T (box at bottom of p.17).

56 T (Just make all the coefficients in the linear combination 0).

57 T (The coefficients of the linear combination all get multiplied by c).

58 T (The coefficients of the linear combination add up).

59 T (By definition of linear combination).

60 T (In order to generate R^m you need at least m vectors, since R^m is m-dimensional space. The simplest generating set is the one of the standard vectors $\mathbf{e_1}, \mathbf{e_2}, \ldots, \mathbf{e_m}\}$. Of course you can add to it more vectors, and it would be still a generating set (adding more vectors can never do harm, but often it does no good).

61 T (Theorem 1.6(a), p. 70)

62 T (p. 48, middle box.

63 T (If the rank of A would have been m, then it would always be consistent, by Theorem 1.6 (p. 70))

64 T (Adding more vectors to a generating set can never do any harm).

Section 1.7 (p. 85)

63. T (If one of the vectors in $\mathcal{S}$ can be expressed as linear combination of other vectors, moving it to the right side would give a way of expressing $\{\mathbf{0}\}$ as a linear combination of members of $\mathcal{S}$ with not-all-zero coefficients).

64 F ('rows' $\rightarrow$ 'columns').

65 F (dependent $\rightarrow$ independent) [Corrected Oct. 8, 2018, thanks to Derek Maciel]

66 T (Since the last column of the augmented matrix is all zeros, translating it to common language means that $x_1 = 0$, $x_2 = 0 \ldots x_n = 0$).

67 T (see box at bottom of p. 83)

68 T (The all-zero solution $x_1 = 0, \ldots, x_n = 0$ is **always** guaranteed to be a (boring, but legal!) solution.

69 F (The simplest example is $x = 0$ (in one variable) whose only solution is $x = 0$. Example in two variables $x_1 + x_2 = 0, x_1 - x_2 = 0$ whose only solution is $x_1 = 0, x_2 = 0$ hence not infinitely many).

70 T (See box at bottom of p. 80).

71 F (It is always **linearly independent** unless $\mathbf{v} = \mathbf{0}$).

72 F (This is only valid if the set has only two vectors in it. For example $\{[0,1]^T, [0,1]^T, [1,1]^T\}$ are linearly dependent (since the third vector is a sum of the first and second, but no vector is a multiple of another one).

73 F (Simplest example, $\{\mathbf{0}\}$ is linearly dependent,but has only one member).

74 T (The rank can be at most 3 [since there are only 3 rows], but in order to be linearly independent the rank must he 4, see box at bottom of p. 83)

75 T (By definition of *homogeneous* system).

76 T (The largest size of an independent set of vectors in R^n is its dimension n).

77 F ('column' $\rightarrow$ 'row').

78 T (The can never be all-zero row in the coefficient part, so you can never have zero=NotZero).

79 F (This is **always** true regardless of the set).

80 T ($1 \cdot \mathbf{0} + \ldots = \mathbf{0}$, in other words $\mathbf{0}$ is always kickable).

81 T (For example, in three dimensions $c_1\mathbf{e_1} + c_2\mathbf{e_2} + c_3\mathbf{e_3} = [c_1, c_2, c_3]^T$ So $c_1\mathbf{e_1} + c_2\mathbf{e_2} + c_3\mathbf{e_3} = [0,0,0]^T$ implies $c_1 = 0, c_2 = 0, c_3 = 0$.)

82 T (no room for more than n linearly independent vectors in n-dimensional space).

2.1 (p. 104)

33 F (only if $m = n$, for example if A and B are 2×3 matrices then AB is nonsense (i.e. undefined).

34 F (a 2×3 matrix times a 3×4 matrix is defined (since $3 = 3$), but the product of a 3×4 matrix by a 2×3 matrix is not defined, since $4 \neq 2$)

35 F (matrix product is usually **not** commutative. Simple example:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad .$$

Then $AB \neq BA$ (you do it!))

36 T (If A is an $n \times n$ matrix then A^2, alias AA is defined since the number of columns of the left A is the same as the number of rows of the second A)

37 F (A and B have to be square matrices of the **same size**)

38 F (The correct version is $(AB)^T = B^T A^T$)

39 T (For example if B is the identity matrix of the same size as A)

40 T (this may be taken as the definition of matrix product)

41 F (the (i, j) entry is $a_{i1}b_{1j} + a_{i2}b_{2j} + \dots a_{ik}b_{kj}$ where k is the number of columns of A (that is the same as the number of rows of B, if AB is defined)

42 F ('... i-th column of A and the j-th row of B' $\rightarrow$ ' ... i-th row of A and the j-th column of B')

43 T (matrix product is **associative** (parentheses do not matter))

44 F (the corrected statement is $(A + B)C = AC + BC$)

45 F (it is much more complicated, see 41)

46 T (Zero times anything is always zero)

47 F. Simple example:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad .$$

48 T (In fact $A_\alpha A_\beta = A_{\alpha+\beta}$)

49 FALSE (The book said True!) ('product of two diagonal matrices' $\rightarrow$ 'product of two diagonal matrices of **the same size**')

50 T (since in the transpose operation, rows become columns and vice versa, so i and j get swapped).

2.3 (p. 131)

33: F (many are not, the most obvious one is the all-zero matrix)

34 T (Def. on p. 122)

35 T (since elementary row operations are reversible)

36 F (' If A and B are matrices ...' $\rightarrow$ (' If A and B are $n \times n$ matrices ...';) [corrected Oct. 9, 2018, thanks to Julia Vida].

37 T (See p. 122 below the definition)

38 T (Since the transpose of an invertible matrix is invertible, and the product of two invertible matrices is invertible)

39 F (see #37)

40 T (since $(A^{-1})^{-1} = A$)

41 T (Theorem 2.2(c))

42 T (Not only that $(AB)^{-1} = B^{-1}A^{-1}$)

43 F ($A^{-1}B^{-1} \rightarrow B^{-1}A^{-1}$)

44 F ('sequence of' $\rightarrow$ one)

45 T (scaling and swapping have n non-zero entries, $r_i + cr_j \rightarrow r_i$ has $n+1$ non-zero entries)

46 F (once you combine two (or more) elementary matrices they are no longer elementary)

47 T (see box at the middle of page 127)

48 T (and to get that matrix, you apply the very same elementary row operation to the identity matrix. See bottom pf p. 126)

49 T (and section 2.4 tells you how to find it)

50 T (follows from the Column Correspondence Principle)

51 F (dependent $\to$ independent)

52 T (follows from the Column Correspondence Principle)

2.4 (p. 143)

35 T (Theorem 2.6 (b))

36 F ('for any two matrices' $\to$ 'for any two $n \times n$ matrices')

37 T (Theorem 2.6)

38 T (by definition of invertible and inverse)

39 T (Theorem 2.6(c))

40 T (also Theorem 2.6(c))

41 T (In fact the reduced-row-echelon form is I_n, and of course has no zero-row, conversely it it had a zero row its rank would be less than n)

42 T (Theorem 2.6(h))

43 T (Theorem 2.6(g))

44 T (Theorem 2.6(g) applied to the transpose of A, that must also be invertible)

45 T (if that happens the rank must be less than n)

46 T (if that happens the rank must be less than n, also when you do matrix multiplication the product the product would have a row of all zeros, hence can't be the identity matrix)

47 T (that's the basis of the algorithm for finding the inverse)

48 F (I_n and $-I_n$ are both invertible, but their sum is the zero matrix, that is clearly not invertible)

49 T (Theorem 2.6 (e) and (f))

50 F ($C = B^{-1}A \to C = A^{-1}B$)

51 F ($B = A^{-1} \to BA = R$)

52 T (B is always a product of elementary matrices, and hence invertible, since an elementary matrix is always invertible, and products of invertible matrices is always invertible)

53 T (top of p. 136)

2.5 (p. 151)

29 T (In AB You break-up A into rows and B into columns)

30 T (box in p. 148)

31 F ($\mathbf{v}$ is a $m \times 1$ matrix for some m and and $\mathbf{w}^T$ is an $1 \times n$ for some n so it is always possible to multiply them getting an $m \times n$ rank-one matrix)

32 T (see 31)

33 F ('For any vectors $\mathbf{v}$, $\mathbf{w}$ ...' $\rightarrow$ ('For any vectors **non-zero** $\mathbf{v}$, $\mathbf{w}$ ...')

34 T (see Eq. (8), p. 149; Book says F, but I disagree)

2.6 (p. 165)

33 F (a matrix has an LU decomposition only if, in the process of getting it to row-echelon form, one does not need to use row-swapping. In other words, if one only uses the elementary row operations $E(i, j; c)$ alias, $cr_j + r_i \rightarrow r_i$).

34 T (The non-zero sub-diagonal entries of L tells you which ones)

35 F ('above and to the right' $\rightarrow$ 'below and to the left')

36 F ($U \rightarrow L$)

37 F ('every matrix' $\rightarrow$ 'every **invertible** matrix')

38 T (p. 157, line 5 from bottom)

39 **Note:** The book has a typo. It probably meant 'c times row j is added to row i'. With this typo corrected, it is still FALSE. $c \rightarrow -c$.

40 **Note:** Same typo as #39. With this typo corrected, it is True.

41 T (see p. 159). (Note that none of the assigned homework problems involve this case, where you first need to permute the rows, but you should know that it is always possible.)

3.1 (p. 209)

45 F (the determinant of a matrix is a **number**)

46 F ($ad + bc \rightarrow ad - bc$)

47 F ('equals zero' $\rightarrow$ 'is **not** equal to zero')

48 F (‘equals zero’ $\rightarrow$ ‘is **not** equal to zero’)

49 T (by expanding with respect to such a row, everything gets multiplied by k)

50 F (the corrected statement is #51)

51 T (p. 202, line above Eq. (3))

52 T (Theorem 3.1, p. 203)

53 F (for large matrices is takes for ever, using the method of 5.2 is much more efficient)

54 T (Use the cofactor expansion and induction)

55 F (e.g. $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$)

56 T (Do cofactor expansion with respect to that row)

57 F (Book says T. The U that shows up in section 2.6 are often not square. What the book probably meant are the Upper triangular matrices that show up in Chapter 3)

58 F (‘left and below’ $\rightarrow$ ‘right and above’)

59 T (In general for an $n \times n$ upper triangular matrix there are at most $(n+1)n/2$ non-zero entries, since $n(n-1)/2$ of them must be 0)

60 T (by definition of transpose)

61 F (‘sum’ $\rightarrow$ ‘product’)

62 T (the determinant of a diagonal matrix is the product of its diagonal entries, and $1^n = 1$)

63 F ($\det[\mathbf{uv}] \rightarrow |\det[\mathbf{uv}]|$ (area is always positive, so we must take the absolute value)

3.2 (p. 220)

39 F (‘squre matrix’ $\rightarrow$ ‘square **upper-triangular** matrix’ also $\rightarrow$ ‘square **lower-triangular** matrix’)

40 T (Th. 3.3.(d), p. 212)

41 F (if the scalar is c the determinants gets multiplied by c)

42 T (Theorem 3.3(a))

43 F (e.g. $A = I_2$ and $B = -I_2$, $\det(A+B) = 0$ but $\det(A) + \det(B) = 2$)

44 T (Theorem 3.4(b), p.214)

45 F ($\det(A) = 0 \to \det(A) \neq 0$)

46 F($\det(A^T) = -\det(A) \to \det A^T = \det A$)

47 T (since $\det A^T = \det A$, and when you transpose a matrix rows become columns, and vice versa)

48 F (if the matrix is invertible, the reduced row echelon form is the identity matrix whose determinant is 1)

49 T (that's the whole point of the determinant, to **determine** whether the matrix is invertible, if $\det(A) \neq 0$ then it is, if $\det(A) = 0$ then it is **not** invertible).

50 T (Theorem 3.2 , p. 205)

51 F (if A is an $n \times n$ matrix, then $\det(cA) = c^n \det(A)$, since every row gets multiplied by c, and since there are n rows, the determinant gets multiplied by c n times, hence c^n)

52 F (Only if the determinant is not 0, Note: Applying Cramer's rule is not required for this class, but you should know about it)

53 T (see Theorem 3.5, in general you need $n + 1$ determinants, since the numerators are all different, but the denominators are the same)

54 T (Theorem 3.4(d), p. 214)

55 T (since 4 is even)

56 F ($\det(-A) = \det(A) \to \det(-A) = -\det(A)$, since 5 is odd)

57 T (follows from Theorem 3.4(b). BTW it is also true for negative integers and for zero)

58 F ($\det(A) = u_{11}u_{22}\cdots u_{nn} \to \det(A) = (-1)^r u_{11}u_{22}\cdots u_{nn}$, where r is the number of row interchanges performed)

4.1 (p. 239)

43 T (part 3 of the definition of 'subspace', bottom of p. 227)

44 T (part 1 of the definition of 'subspace', bottom of p. 227)

45 F ('null space' → '**zero space**')

46 T (Theorem 4.1, p. 231)

47 T (p. 237 above Example 10)

48 F ($R^n \to R^m$) (The columns of an $m \times n$ matrix are vectors with m components, hence belong to R^m, and hence the column space, that is the span of the set of columns, obviously is a subspace of R^m)

49 F ($R^m \to R^n$) (each row of an $m \times n$ matrix is row vector of length n, hence belongs to R^n)

50 F ($\{A\mathbf{v} \,|\, \mathbf{v} \in R^n\} \to \{\mathbf{v}A \,|\, \mathbf{v} \in R^m\}$)

51 T (when you transpose a matrix, rows become columns and vice versa)

4.2 (pp. 251-252)

33 F (it has **infinitely many** bases, but they all have the same number of elements, called the *dimension*)

34 T (not only that, it has many of them (see 33))

35 F (large $\to$ small)

36 T (Of course $\mathcal{S}$ is a generating set of $Span\mathcal{S}$, and if it happens to be also linearly independent, it is a basis, by definition of basis)

37 T (Theorem 4.3, the *Reduction Theorem*)

38 T (but the phrasing is confusing)

39 T (and that common number of elements is called the *dimension*)

40 F ('columns' $\to$ '**pivot** columns')

41 F (the pivot columns of the **original matrix** form a basis, i.e. the columns corresponding to the pivot columns of the rref)

42 T (but strictly speaking the book should have said 'the set of vectors', not just 'the vectors')

43 F ('exactly' $\to$ 'at least')

44 T (corrected form of 43)

45 T (Since $\mathcal{S}$ has k vectors it is also a generating set, hence it is both linearly independent and a generating set hence, by definition a basis)

46 T (The maximum size of a linearly independent set is the dimension of the subspace)

47 T (The standard basis $\{\mathbf{e_1}, \ldots, \mathbf{e_n}\}$ has n members),

48 T (by definition of *standard basis*)

49 T (Theorem 4.4, *'The Extension Theorem'*)

50 F (most subspaces don't)

4.3 (pp. 261)

41 F (There are lots and lots of different subspaces with the same dimension k, if $k < n$.)

42 T (R^n only has one subspace of dimension n, itself!)

43 T

44 F (rank $\rightarrow$ nullity)

45 F (nullity $\rightarrow$ rank)

46 T (p. 258, top box)

47 T (also true if you replace 'reduced row echelon from' by 'row echelon form)

48 F (you need to look at the correponding columns)

49 T (elementary row operations preserve the null space)

50 F ('non-zero rows of a matrix' $\rightarrow$ 'non-zero rows of its [reduced] row-echelon form')

51 F ('first k rows of A' $\rightarrow$ "first k rows of the reduced-row echelon form of A')

52 F (the row space is a subspace of R^n, the column space is a subspace of R^m)

53 T (famous theorem; both equal the rank)

54 T (famous theorem)

55 F (nullity of A is $n - rank(A)$, nullity of A^T is $m - rank(A)$, unless $m = n$, they are different)

56 F ($m \rightarrow n$)

57 T (corrected form of #56)

5.1 (p. 300)

41 F ('some vector $\mathbf{v}$' $\rightarrow$ ('some **non-zero** vector $\mathbf{v}$'. [$A\mathbf{0} = \lambda\mathbf{0}$ is always true])

42 F(same as in #41)

43 T (definition of eigenvalue)

44 T (if $\mathbf{v}$ is an eigenvector, it means that $A\mathbf{v}$ is a multiple of $\mathbf{v}$, and their ratio can only be one thing)

45 T (every multiple of an eigenvector is yet-another-eigenvector. Indeed if $A\mathbf{v} = \lambda\mathbf{v}$ then for any non-zero number k $Ak\mathbf{v} = \lambda(k\mathbf{v})$)

46 F ('column space' $\to$ 'nullspace')

47 T (striclty speaking, in this class we don't talk about 'linear operators' but the linear operator cosseponding to a matrix A is simply the operation $\mathbf{x} \to A\mathbf{x}$)

48 T

5.2 (pp. 310-311)

53 F (eigenvectors $\to$ eigenvalues)

54 T (the eigenvalues are the roots of the characteristic polynomial)

55 T

56 F (the rref of an invertible matrix is the identity matrix whose only eigenvalue is 1)

57 F (the rref of an invertible matrix is the identity matrix whose eigenspace is) he whole of R^n)

58 F (the characteristic equation may have multiple roots)

59 F (it may be complex)

60 T (but only if real numbers are also considered complex, as they should)

61 F (some (or all) of them may be complex)

62 T (If you allow repeats)

63 T

64 F ('equal' $\to$ 'larger-or-equal to', by theorem 5.1)

65 T (By theorem 5.1 it is ≤ 1, but it can't be 0-dimensional)

66 F (the book is talking about a general matrix, with possibly complex entries. In this class we only do real matrices)

67 T (since the characteristic equation has real coefficients)

68 T (the degree of the characteristic polynomial is n so the sum of the multipliticities of its roots must be n)

69 T (The characteristic equation of I_n is $(1-t)^n$)

70 T (The characteristic equation of I_n is $(1-t)^n$)

71 F (it has exactly one eigenvalue, namely 0 with multiplicity n)

72 T (since these are the values for which $A - tI_n$ is **not** invertible.)

5.3 (pp. 322)

29 F (e.g. $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is not. More generally if there is an eigenvalue whose multiplicity is strictly more than the dimension of its eigenspace, then the matrix is **not** diagonalizable)

30 T (and then the matrix P can be made by using this basis as its columns)

31 T (see #30.)

32 T (p. 318, blue box)

33 F (You can permute the eigenvalues in every way you want, and still get an OK D.)

34 F ('distinct eigenvectors' → 'distinct **eigenvalues**')

35 F (It may have multiplicities. For example, the epitome of a diagonal matrix, the identity matrix I_n has only **one** eigenvalue (1) with multiplicity n.)

36 T (a famous theorem.)

37 F (the sum of the multiplicities is **always** n, but not every square matrix is diagonalizable. **Important Note**: The above corresponds to **all** eigenvalues. If you only care about real eigenvalues, then **37** is all the more False.)

38 T (if you allow (as you should!) complex eigenvalues). If you only care about real eigenvalues (like in this stupid class), then it is False.

39 T (if it can be diagonalized, then the dimension of the eigenspace equals the multiplicity for each of the eigenvalues.)

40 F ('rank' → 'nullity'.)

41 F (e.g. I_n only has **one** eigenvalue (1) with multiplicity n).

42 T (Of course, take $P = I_n$).

43 T (e.g. for the 2×2 case,

$$\begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} d_1 \\ 0 \end{bmatrix} = d_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad ,$$

so an eigenvector corresponding to d_1 is $\mathbf{e_1}$, and

$$\begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ d_2 \end{bmatrix} = d_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad ,$$

so an eigenvector corresponding to d_2 is $\mathbf{e_2}$.)

44 F ($PAP^{-1} \rightarrow P^{-1}AP$.)

45 F (if you take any eigenvector and multiply it by any non-zero scalar, you get yet another eigenvector, and of course they are not linearly independent.)

46 T (famous theorem)

47 F (the linear factors can have repeats.)

48 T (If you are willing to accept complex eigenvalus and eigenvectors. Otherwise F).

6.1 (pp. 372)

61 T

62 F (the dot product of two vectors in R^n is a **number**.)

63 F (‘norm’ $\rightarrow$ ‘norm-sqared’.)

64 F (‘multiple’ $\rightarrow$ ‘positive multiple’.)

65 F (the norm of a sum of vectors is $\leq$ then the sum of the norms.)

66 T (The Pythagorean theorem for R^n.)

67 T

68 T (norm is the square-root of a sum of squares, hence must be postive or zero.)

69 T (in other words, if $\mathbf{v}$ is **not** the zero-vector, then its norm is positive.)

70 F (If $\mathbf{u}\dot{\mathbf{v}} = 0$ then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, but usually not zero.)

71 F (‘$=$’ $\rightarrow$ ‘$\leq$’.)

72 T (Th. 6.1(c) [p. 364].)

73 T (the definition of distance.)

74 T (Th. 6.1(f) [p. 364].)

75 T (Th. 6.1(d) [p. 364].)

76 F (the corrected statement is $A\mathbf{u}.\mathbf{v} = \mathbf{u}.A^T\mathbf{v}$.)

77 T (since $|-1| = 1$).

78 F (the corrected statement is ($||u + v||^2 = ||u||^2 + ||v||^2$.)

79 T (p. 366)

80 T

6.2 (pp. 386)

41 T (Theorem 6.5).

42 T (use Gram-Schmidt).

43 F ('single vector' $\rightarrow$ '**non-zero** single vector'.

44 T (since they are linearly independent).

45 T (bottom of p. 376; Amazing short-cut).

46 T (it is called the normalization of $\mathbf{u}$.

47 T (they all have unit norm, and of course $\mathbf{e_i}.\mathbf{e_j} = 0$.

48 T (like 41, orthonormal is a bigger deal than orthogonal, so of course it is also true)

49 F (of course not, there are not orthogonal to each other)

50 F (Even in R^3, any two vectors in the xy-plane are both orthogonal (i.e. perpendicular) to $\mathbf{e_3}$ [alias $\mathbf{k}$], without (usually) being perpendicular to each other.).

51 T (That is its goal in life!)

52 F (Only R is).

6.4 (pp. 410)

28 F ('the sum' $\rightarrow$ 'the sum of the squares')

29 T (top of p. 404)

30 F (It can fit data with quadratic curves, cubic curves, etc. The matrix C is then no longer $n \times 2$ but, resepectively, $n \times 3$, $n \times 4$, etc.)

THE END
