**Solutons to the REAL Quiz #4 (Dr. Z., Math 250)**

**1.** (3 pts.) Find an elemenary matrix $E$ such that $EA = B$, where $A$ and $B$ are as follows:

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 3 & -1 & 0 \\ -1 & 1 & 6 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 1 & 2 & -2 \\ 3 & -1 & 0 \\ 0 & 3 & 4 \end{bmatrix}$$

**Sol. of 1**: We have to decide which **elementary row operation**, when applied to $A$ yields $B$. The first two rows are exatly the same, so this means that $r_3$ has been changed. Obviously the new $r_3$ is not a multiple of the old one, so this means that the new $r_3$ is the old $r_3$ plus (or minus) a multiple of either $r_1$ or $r_2$. Since the difference between the new $r_3$ and the old $r_3$ is $[1, 2, -2]$, that is exactly $r_1$, the elementary row operation that we need is $r_3 + r_1 \to r_3$.

Now we apply this **very same** elementary row operation to the identity matrix, getting

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} r_3 + r_1 \to r_3 \\ \to \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad .$$

This is the desired matrix $E$. You are welcome to check, by doing the matrix multiplication, that indeed $EA = B$.

**Ans.**:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad .$$

**Comment**: About %70 of the people got it right. The rest did a much more difficult problem converting $A$ to reduced-row-echelon-form. **PLEASE** read the question!

**2.** (3 pts.) Determine whether the following matrix is invertible, and if it is, finds its inverse

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$$

**Sol. of 2**: We first bring it to **row-echelon form**.

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \begin{matrix} r_2 - (3/2)r_1 \to r_2 \\ \to \end{matrix} \begin{bmatrix} 2 & 3 \\ 0 & 1/2 \end{bmatrix}$$

So we know that it is invertible, and we must go on, all the way. Continuing

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \begin{matrix} r_2 - (3/2)r_1 \to r_2 \\ \to \end{matrix} \begin{bmatrix} 2 & 3 \\ 0 & 1/2 \end{bmatrix} \begin{matrix} r_1 - 6r_2 \to r_1 \\ \to \end{matrix} \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{matrix} (1/2)r_1 \to r_1, 2r_2 \to r_2 \\ \to \end{matrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad .$$

Now we have to **mimick** the same sequence operations starting with the **identity matrix**, $I_2$:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{matrix} r_2 - (3/2)r_1 \to r_2 \\ \to \end{matrix} \begin{bmatrix} 1 & 0 \\ -3/2 & 1 \end{bmatrix} \begin{matrix} r_1 - 6r_2 \to r_1 \\ \to \end{matrix} \begin{bmatrix} 10 & -6 \\ -3/2 & 1 \end{bmatrix} \begin{matrix} (1/2)r_1 \to r_1, 2r_2 \to r_2 \\ \to \end{matrix} \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} \quad .$$

This is the **ans.**.

**Ans. to 2**: The matrix is invertible, and its inverse is:

$$\begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} \quad .$$

**Comment**: About %80 of the people got it right completely.

**3.** True or False (Explain when appropriate)

(a) (1 pt.) If a square matrix has a column consisting of all zeros, then it must be invertible.

**Sol. of 3(a)**: **False**. If it has a column of all zeroes, then it is never invertible (For an $n \times n$, the rank is less than $n$, so its reduced row echelon form will never be $I_n$).

(b) (1 pt.) The pivot columns of a matrix are linearly independent.

**Sol. of 3(b)**: **True**. The pivot columns of the reduced-row-echelon-form are (different) standard vectors, and obviously linearly independent. By the **column-correspondence property** the same applies to the pivot columns of the original matrix.