SOLUTIONS to the REAL Quiz #2 (Dr. Z., Math 250)

1. (5 pts.) Determine whether the given system is consistent, and if so, find its general solution.

$$x_1 - x_2 + x_4 = -4$$

$$x_1 - x_2 + 2x_4 + 2x_5 = -5$$

$$3x_1 - 3x_2 + 2x_4 - 2x_5 = -11 \quad .$$

Sol. of 1: The **augmented** matrix of the system is (with variables x_1, x_2, x_3, x_4, x_5) is

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 & -4 \\ 1 & -1 & 0 & 2 & 2 & -5 \\ 3 & -3 & 0 & 2 & -2 & -11 \end{bmatrix} \quad .$$

We must bring it first to **row echelon form**. If we are lucky we'll get a row of 0's except for a non-zero entry at the end, and we would conclude that the system is inconsistent. If we are not lucky, and we won't get such a scenario, then we must go on to phase II and get it in **reduced row echelon form**, so that we can find the general solution.

Doing $r_2 - r_1 \to r_2$ and $r_3 - 3r_1 \to r_3$ yields

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & -1 & -2 & 1 \end{bmatrix} \quad .$$

the pivot on the second row is at column 4 (the $(2,4)$ entry) and we want to make 0 everything below it. Doing $r_2 + r_3 \to r_3$ gives

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad .$$

This is in **row-echelon** form, and since we don't have an all-0-row-followed-by-non-zero (we have a complete 0 row, but that's OK), we conclude that the system is **consistent**, and unfortunately, we must proceed to the next phase.

Doing $r_1 - r_2 \to r_1$ yields

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -2 & -3 \\ 0 & 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad .$$

This is in **reduced-row-echelon form**. In everyday language this means

$$x_1 - x_2 - 2x_5 = -3 \quad ,$$

$$x_4 + 2x_5 = -1 \quad .$$

The **basic variables** (corresponding to the pivot columns 1 and 4) are x_1 and x_4 and the **free variables** are the remaining variables x_2, x_3, x_5. The general solution is

$$x_1 = -3 + x_2 + 2x_5 \quad , \quad x_2 = x_2 \quad , \quad x_3 = x_3 \quad ,$$

$$x_4 = -1 - 2x_5 \quad , \quad x_5 = x_5 \quad .$$

Finally the general solution in **vector form** is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3 + x_2 + 2x_5 \\ x_2 \\ x_3 \\ -1 - 2x_5 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} .$$

2. True or False (Explain when appropriate)

(a) (1 pt.) The number of pivot columns of a matrix equals its rank.

Sol. of 2a): True (by definition, pivot columns correspond to pivot entries, and their number is the number of non-zero rows in the reduced row echelon form.)

(b) (1 pt.) If a matrix is in row echelon form, the leftmost non-zero entry of each non-zero row must be 1 .

Sol. of 2b: False. The corrected statement is

"If a matrix is in **reduced** row echelon form, the leftmost non-zero entry of each non-zero row must be 1" .

(c) (1 pt.) A system of linear equations is called inconsistent if it has no solutions.

Sol. of 3b: True (by definition of consistent)