Solutions to Dr. Z.'s Math 250 REAL Quiz #1

1. (3 pts.) Compute the matrix-vector product

$$\begin{bmatrix} 2 & -3 \\ -4 & 5 \\ 11 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ 2a \end{bmatrix} \quad ,$$

where a is a real number.

Solution

$$\begin{bmatrix} 2 & -3 \\ -4 & 5 \\ 11 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ 2a \end{bmatrix} = a \begin{bmatrix} 2 \\ -4 \\ 11 \\ 0 \end{bmatrix} + 2a \begin{bmatrix} -3 \\ 10 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2a \\ -4a \\ 11a \\ 0 \end{bmatrix} + \begin{bmatrix} -6a \\ 10a \\ -2a \\ 2a \end{bmatrix}$$

$$= \begin{bmatrix} 2a - 6a \\ -4a + 10a \\ 11a - 2a \\ 0 + 2a \end{bmatrix} = \begin{bmatrix} -4a \\ 6a \\ 9a \\ 2a \end{bmatrix} \quad .$$

Ans. to 1:

$$\begin{bmatrix} -4a \\ 6a \\ 9a \\ 2a \end{bmatrix} \quad .$$

2. (3 pts.) If possible, write the vector

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad ,$$

as a linear combination of the vectors in $\mathcal{S}$, where

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} , \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \right\} \quad .$$

Sol. of 2: We are looking for numbers c_1 and c_2 such that

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \quad .$$

Spelling it out:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 + 3c_2 \\ 2c_1 + 4c_2 \\ 3c_1 + 5c_2 \end{bmatrix} \quad .$$

This means that we have to solve the system of **three** equations with **two** unknowns, c_1, c_2

$$(i)\, c_1 + 3c_2 = 1 \quad , \quad (ii)\, 2c_1 + 4c_2 = 1 \quad , \quad (iii)\, 3c_1 + 5c_2 = 1 \quad .$$

From the first equation, $c_1 = 1 - 3c_2$. Plugging into the second, we get $2(1 - 3c_2) + 4c_2 = 1$, so $2 - 6c_2 + 4c_2 = 1$ so $-2c_2 = -1$ giving $c_2 = \frac{1}{2}$. Pluging into $c_1 = 1 - 3c_2$ we get $c_1 = 1 - 3 \cdot \frac{1}{2} = -\frac{1}{2}$. We have to make sure that (iii) is correct, but it is. Indeed

$$3(-\frac{1}{2}) + 5(\frac{1}{2}) = 1 \quad .$$

So $c_1 = -\frac{1}{2}$ and $c_2 = \frac{1}{2}$.

Ans. to 2:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{-1}{2} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \quad .$$

3. True or False (Explain when appropriate)

(a) (1 pt.) The coefficients in a linear combination can always be chosen to be positive numbers.

Ans. to 3(a): False. They are often negative. For example the **only way** to express

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad ,$$

as a linear combination of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1) \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad .$$

Here one of the coefficients is negative.

Even worse: The only way to express

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad ,$$

as a linear combination of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} = (-1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-1) \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad .$$

Here both of the coefficients are negative.

(b) (1 pt.) If A is an $m \times n$ matrix, then the only vector $\mathbf{u}$ in R^n such that $A\mathbf{u} = \mathbf{0}$ is $\mathbf{u} = \mathbf{0}$.

Ans. to 3(b): False. For example, if

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$A\mathbf{u} = \mathbf{0}$ and $\mathbf{u}$ is **not** $\mathbf{0}$.