Solutions to the Attendance Quiz for Lecture 14

1. Find a generating set for the following subspace

$$\{\begin{bmatrix} r-s+3t \\ 2r-t \\ -r+3s+2t \\ -2r+s+t \end{bmatrix} \in R^4 : r, s, \ and\ t, \quad are \quad scalars\}$$

Sol. of 1: We reexpress the general member of this subspace as a linear combination of numerical vectors by taking the parameters r, s, t outside:

$$\begin{bmatrix} r-s+3t \\ 2r-t \\ -r+3s+2t \\ -2r+s+t \end{bmatrix} = r\begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 3 \\ 1 \end{bmatrix} + t\begin{bmatrix} 3 \\ -1 \\ 2 \\ 1 \end{bmatrix} \quad ,$$

this is a typical **linear combination** of the vectors in the set

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix} \quad , \quad \begin{bmatrix} -1 \\ 0 \\ 3 \\ 1 \end{bmatrix} \quad , \quad \begin{bmatrix} 3 \\ -1 \\ 2 \\ 1 \end{bmatrix} \right\} \quad .$$

This is the **answer**.

Comment: About %70 of the people got it right.

2. Find a generating set for the null space of the matrix

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Sol. to 1. We have to find the general solution of the system $A\mathbf{x} = \mathbf{0}$, where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

We bring it to **reduced-row-echelon form**.

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \xrightarrow{r_3 - r_1 \to r_3} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix} \xrightarrow{r_3 - 2r_2 \to r_3} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{r_1 + 2r_2 \to r_1}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-r_2 \to r_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad .$$

Now it is in **reduced-row-echelon-form**.

The **pivot columns** are the first and the second, and the third column is a **free-column** corresponding to the free variable x_3.

In everyday notation this system is (with null-space the right hand-side vector is always $\mathbf{0}$):

$$x_1 + 2x_3 = 0 \quad ,$$

$$x_2 - x_3 = 0 \quad ,$$

Solving we get

$$x_1 = -2x_3$$

$$x_2 = x_3$$

$$x_3 = x_3$$

where x_3 is **free** (i.e. an *arbitrary* real number). In vector notation, the general solution is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \quad .$$

So a generating set is the one-element set

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\} \quad .$$

This is a correct **answer**.

Comment: About %40 of the people got it completely right. But quite a few people gave the answer in *wrong format*, and committed a serious *conceptual error*.

The output is a **set**, not a vector.

Examples of wrongly-formatted answers:

Wrong Answer 1:

$$\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

Why is it wrong?: Because this is a vector! The answer is a *set* of vectors, that in this case happens to be a set with one element.

Wrong Answer 2:

$$x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

Why is it wrong?: Because this is a general solution of the system $A\mathbf{x} = \mathbf{0}$. It is an important step, but the format is wrong. First only look what is next to x_3 and then add the $\{\}$ to make it a set.

Note: There are many other correct solutions. For example

$$\left\{ \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \right\} \quad .$$