Solutions to Attendance Quiz for Lecture 12

1. Compute the determinant of the following matrix along the second row.

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & -1 & 4 \\ -2 & 1 & 2 \end{bmatrix}$$

Sol. to 1:

$$\det \begin{bmatrix} 2 & -1 & 1 \\ 0 & -1 & 4 \\ -2 & 1 & 2 \end{bmatrix} = -(0) \det \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} + (-1) \begin{bmatrix} 2 & 1 \\ -2 & 2 \end{bmatrix} - (4) \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$$

$$= 0 + (-1)((2)(2) - (1)(-2)) - 4((2)(1) - (-1)(-2) = 0 - 6 + 0 = -6 \quad .$$

Ans. to 1: -6.

Comment: About %60 of the people got it right. Most other people did it the right way but messed up with the calculations. Please be very careful with the signs and don't trust mental math. Do everything on paper.

2. Find the value of c for which the matrix is not invertible

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & -1 & c \\ -2 & 1 & 2 \end{bmatrix}$$

Sol. to 2: We could expand it with respect to the first row, but since there is a 0 at the second row, it is more efficient to do it with respect to the second row, like in the above problem.

$$\det \begin{bmatrix} 2 & -1 & 1 \\ 0 & -1 & c \\ -2 & 1 & 2 \end{bmatrix} = -(0) \det \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix} + (-1) \begin{bmatrix} 2 & 1 \\ -2 & 2 \end{bmatrix} - (c) \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$$

$$= 0 + (-1)((2)(2) - (1)(-2)) - c((2)(1) - (-1)(-2) = 0 - 6 + 0 = -6 \quad .$$

This does not depend on c! Since -6 is **not** 0, this means that this matrix is **always** invertible, regardless of the value of c. So the problem lied when it asked you to find "the value" of c for which the matrix is not invertible. There is no such c!

Ans. to 2: There is no such c.

Comments: **1.** The problem was confusing. It should have said, "Find the value of c, if any, for which ...", or "find the set of values of c for which ...", then the answer would be the empty set.

2.: Only %25 of the people got it right. Most other people were on the right track. Some were confused by the wording of the problem.