NAME: (print!) ____

E-Mail address: ____________________

MATH 250 (2), Dr. Z. , Exam 1, Thurs., Oct. 11, 2018, 10:20-11:40am, TIL-258

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM

No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.

Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

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tot.: (out of 100)

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1. (10 pts. altogether)

(a) (7 pts.) Determine if the given system of equations in the variables x_1, x_2, x_3, x_4 is consistent, and if so, find its general solution.

$$x_2 + x_4 + x_1 + x_3 = 4 \quad ,$$

$$x_2 + x_3 + x_1 = 2 \quad .$$

(b) (3 points) Express the general solution of part a) in **vector notation**.

2. (10 pts.) Let

$$\mathcal{S} = \left\{ \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 5 \end{bmatrix} \right\},$$

determine whether the set $\mathcal{S}$ is linearly independent or linearly dependent. In case it is linearly dependent, write the zero vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ explicitly as a non-trivial linear combination of the vectors in $\mathcal{S}$.

3. (10 pts altogether) Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad , \quad C = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Calculate the following matrix products, if they are defined, or explain why they don't make sense.

(a) (4 points) BB^T

(b) (2 points) BA

(c) (4 points) C^3

4. (10 pts.) For the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

compute the matrix A^{100007}.

5. (10 pts.) Find the rank (8 pts) and nullity (2 pts) of the following matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 0 & -4 & 1 \end{bmatrix} \quad .$$

6. (10 pts altogether) Find an elementary matrix E such that $EA = B$.

(a) (4 pts)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 3 & 2 \\ 2 & -1 & 0 & 4 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 3 & 2 \\ 0 & -5 & -6 & -4 \end{bmatrix}$$

(b) (3 pts)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 3 & 2 \\ 2 & -1 & 0 & 4 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 2 & -1 & 0 & 4 \\ -1 & 1 & 3 & 2 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

(c) (3 pts)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 3 & 2 \\ 2 & -1 & 0 & 4 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & -1 & -3 & -2 \\ 2 & -1 & 0 & 4 \end{bmatrix}$$

7. (10 pts.) Find $(ABC)^{-1}$ if

$$A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad , \quad B^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \quad , \quad C^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

8. (10 pts. altogether , 2 each) **True** or **False**? Give a short explanation (if applicable), or give an example for which it is false.

(a) Every system of linear equations has at least one solution.

(b) If a matrix is in reduced row-echelon form then the pivot entry of each pivot-column must be 1 .

(c) If A is an $m \times n$ matrix, then $A\mathbf{x} = \mathbf{b}$ is consistent for every b in R^m if and only if the rank of A is m .

(d) If A and B are invertible $n \times n$ matrices then $A + B$ is invertible.

(e) Every column of a matrix is a linear combination of its pivot columns.

9 (10 pts. altogether , 2 each) **True** or **False**? Give a short explanation (if applicable), or give an example for which it is false.

(a) The transpose of a matrix is a matrix of the same size.

(b) If A is a matrix for which the sum $A + A^T$ is defined, then A is a square-matrix.

(c) If A is a matrix for which the product $A\,A^T$ is defined, then A is a square-matrix.

(d) Every vector in R^3 is a linear combination of the standard vectors $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ (these are called **i**, **j**, and **k** in Physics (and calc3))

(e) If A, B and C are invertible matrices, then $(ABC)^{-1} = A^{-1}B^{-1}C^{-1}$

10 (10 pts) Use any method to solve the following system of linear equations, in the variables x_1, x_2, x_3, x_4, x_5.

$$x_1 + x_2 + x_3 + x_4 + x_5 = 5 \quad , \quad x_1 + x_2 + x_3 + x_4 = 4 \quad ,$$

$$x_1 + x_2 + x_3 = 3 \quad , \quad x_1 + x_2 = 2 \quad , \quad x_1 - x_2 = 0 \quad .$$