Maple Challenges for Dr. Z.'s Math250(2) Fall 2018 class

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Note: To get a random $m \times n$ matrix in Maple, type:

```
with(linalg):  A:=randmatrix(m,n);
```

Then you can experiment with it.

1: By using `randmatrix` give 20 examples of 5×5 matrices. For each of them find the **reduced-row-echeclon** form. Out of these 20 examples, how many turned out to be the 5×5 identity matrix, I_5?

2: By using `randmatrix` give 20 examples of 2×5 matrices. For each of them find the **reduced-row-echeclon** form. Out of these 20 examples, how many have the first two columns the standard vectors $\mathbf{e_1}, \mathbf{e_2}$?

3. In order to find out how long a Maple command takes, type

`time(command)`.

For example, to find out how long Maple takes to factorize $2^{198} - 1$, type

`time(ifactor(2**98-1));` .

The **permanent** of a matrix is similar to the determinant, but in the *cofactor expansion* all the signs are plus. For example

$$permanent \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ab + cd$$

$$permanent \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \cdot permanent \begin{bmatrix} e & f \\ h & i \end{bmatrix} + b \cdot permanent \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \cdot permanent \begin{bmatrix} d & e \\ g & h \end{bmatrix} \quad .$$

The Maple command, for finding the permanent is (assuming that you have already typed `with(linalg);`) :

`permanent(A);` ,

and to find the time it takes, do

```
time(permanent(A));   ,
```

While Maple can very fast (0.12 seconds) find the determinant of a 30×30 matrix (using row-and-column operations), and even (3 seconds), the determinant of a 100×100 matrix, it can't do the same for the permanent. You would be both rich and famous if you can find an algorithm for computing in less than one day of computer time, the permanent of an arbitrary 100×100 matrix.

Part a:

By typing:

```
with(linalg):  A:=randmatrix(15,15):  time(det(A)); time(permanent(A)); ,
```

ten times, find out for each, how longs it takes to compute the determinant and how long it takes to compute the permanent for random 15×15 matrices.

Part b:

By typing:

```
with(linalg):  A:=randmatrix(17,17):  time(det(A)); time(permanent(A)); ,
```

ten times, find out for each, how longs it takes to compute the determinant and how long it takes to compute the permanent for random 17×17 matrices.

Part c:

By typing:

```
with(linalg):  A:=randmatrix(20,20):  time(det(A)); time(permanent(A)); ,
```

ten times, find out for each, how longs it takes to compute the determinant and how long it takes to compute the permanent for random 20×20 matrices.

Warning: it will take awhile! (if it does not stop in ten minutes, you can abort it by clicking on the stop icon).