NAME: (print!) ____

E-Mail address: _________________________________ SCC: (please circle): Yes/No

MATH 250 (2), Dr. Z. , Final Exam , Tue., Dec. 18, 2018, TIL-258, 12:00-3:00pm

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question)

Show all your work! No calculators, no cheatsheets

CHECK ALL YOUR ANSWERS! (whenever applicable) .

Do not write below this line

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1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)
11. (out of 10)
12. (out of 10)
13. (out of 10)
14. (out of 10)
15. (out of 10)
16. (out of 10)
17. (out of 10)
18. (out of 10)
19. (out of 10)
20. (out of 10)

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tot. (out of 200)

Reminders:
Least Squares: $y = a_0 + a_1 x$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = (C^T C)^{-1} C^T \mathbf{y} \quad , \quad C = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ . & . \\ . & . \\ . & . \\ 1 & x_n \end{bmatrix} \quad , \quad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{bmatrix} \quad .$$

Gram-Schmidt: If $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ is a basis for a subspace W, an orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is as follows:

$$\mathbf{v}_1 = \mathbf{u}_1 \quad ,$$

$$\mathbf{v}_2 = \mathbf{u}_2 - \frac{\mathbf{u}_2 \cdot \mathbf{v}_1}{||\mathbf{v}_1||^2}\mathbf{v}_1 \quad ,$$

$$\mathbf{v}_3 = \mathbf{u}_3 - \frac{\mathbf{u}_3 \cdot \mathbf{v}_1}{||\mathbf{v}_1||^2}\mathbf{v}_1 - \frac{\mathbf{u}_3 \cdot \mathbf{v}_2}{||\mathbf{v_2}||^2}\mathbf{v}_2 \quad ,$$

etc.

Orthonormal basis: $\mathbf{w}_i = \frac{\mathbf{v}_i}{||\mathbf{v}_i||}$

QR: $\quad Q = [\,\mathbf{w}_1 \quad \mathbf{w}_2 \quad \mathbf{w}_3 \quad \cdots],$

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \ldots \\ 0 & r_{22} & r_{23} & \ldots \\ 0 & 0 & r_{33} & \ldots \\ \ldots & & & \end{bmatrix}$$

where

$$r_{ij} = \mathbf{w_i} \,.\, \mathbf{a_j}$$

$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64, 9^2 = 81,\ 10^2 = 100$
$11^2 = 121, 12^2 = 144, 13^2 = 169, 14^2 = 196, 15^2 = 225, 16^2 = 256, 17^2 = 289, 18 = 324, 19^2 = 361,\ 20^2 = 200$

1. (10 points) Find the least-square line that best fits the following set of points

$$\{(0,0),(1,0),(2,6)\}$$

Ans.:

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2. (10 points) Find the QR decomposition of the matrix

$$\begin{bmatrix} 5 & 22 \\ -12 & -19 \end{bmatrix} \quad .$$

Ans.: $Q =$ $R =$

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3 (10 points) Use the LU method (no credit for other methods!) to solve the following system of linear equations:

$$x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1 + 2x_2 + 2x_3 + 2x_4 = 7$$

$$x_1 + 2x_2 + 3x_3 + 3x_4 = 9$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 10$$

Ans.: $x_1 =$ $x_2 =$ $x_3 =$ $x_4 =$

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4. (10 points) By viewing the two 6×6 matrices below as 3×3 block matrices whose entries are certain 2×2 matrices, (that you have to decide on), use the method of partitioning (No credit for other methods!) to do the following matrix product. Explain everything!

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Ans.:

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5. (10 points) Suppose that $\mathbf{u}$,$\mathbf{v}$ are vectors in R^{1001} such that

$$||\mathbf{u}|| = 1 \quad , \quad ||\mathbf{v}|| = 1 \quad ,$$

$$\mathbf{u} \cdot \mathbf{v} = -\frac{1}{6} \quad ,$$

Compute:

$$||\mathbf{u} + 3\,\mathbf{v}|| \quad .$$

Ans.:

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6. (10 points) Find, if possible, an invertible matrix P and a diagonal matrix D such that

$$PDP^{-1} = \begin{bmatrix} -2 & 2 \\ -10 & 7 \end{bmatrix} \quad ,$$

or explain why it is not possible.

Ans.: $D =$ $\qquad\qquad\qquad\qquad$ $P =$

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7. (10 points, 2.5 each) Complete the following sentences

a: A vector $\mathbf{u}$ in R^n is a **linear combination** of the set $\mathcal{S} = \{\mathbf{u_1} \quad , \quad \ldots \quad , \quad \mathbf{u_k}\}$ if ...

b: A set of vectors $\mathcal{S} = \{\mathbf{u_1} \quad , \quad \ldots \quad , \quad \mathbf{u_k}\}$ is **linearly independent** if ...

c: A set of vectors $\mathcal{S} = \{\mathbf{u_1} \quad , \quad \ldots \quad , \quad \mathbf{u_k}\}$ is a **generating set** for a subspace V of R^n if ...

d: A set of vectors $\mathcal{S} = \{\mathbf{u_1} \quad , \quad \ldots \quad , \quad \mathbf{u_k}\}$ is a **basis** for a subspace V of R^n if ...

8. (10 points) Show that $\lambda = 2$ is an eigenvalue of the matrix

$$\begin{bmatrix} 5 & 3 & 9 \\ 3 & 5 & 9 \\ -3 & -3 & -7 \end{bmatrix}$$

and determine a basis for its eigenspace.

Ans.:

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9. (10 points, 2.5 each) Complete the following sentences
a: An **eigenvalue** of a square $(n \times n)$ matrix A, is a number t such that ...

b: An **eigenvector** of a square $(n \times n)$ matrix A is a vector $\mathbf{x}$ in R^n such that ...

c: A **pivot entry** in the row-echelon (or reduced-row-echelon) form of matrix is an entry that is ...

d: An **elementary row operation** is one of the following operations involving either one or two rows of a matrix: ...

10. (10 points) Find a basis for the null space of the 2×5 matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad .$$

Ans.:

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11. (10 points altogether, 2 each) Determine whether the following statements are True or False. In case it is false, give a corrected version.

(a) The subspace $\{\mathbf{0}\}$ is called the null-space.

(b) If $\mathcal{S}$ is a linearly independent subset and Span $\mathcal{S}$=V, then $\mathcal{S}$ is a basis for V.

(c) R^9 contains exactly nine different subspaces.

(d) A vector $\mathbf{v}$ is in $ColA$ if and only if $A\mathbf{x} = \mathbf{v}$ is consistent.

(e) The pivot columns of the matrix A (i.e. the columns corresponding to the pivot columns of the reduced-row-echelon form R, but those of A, not of R) always form a basis for its column space.

12. (10 points) Suppose that A and B are square matrices such that

$$\det A = 2 \quad , \quad \det B = -1 \quad .$$

Find $\det(A^3 B^{-100} A^{-2})$.

Ans.:

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13. (10 points) For what value(s) of c is the following matrix

$$\begin{bmatrix} c & 1 & 1 \\ 1 & c & 1 \\ 1 & 1 & c \end{bmatrix}$$

is **not** invertible.

Ans.:

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14. (10 points)
Determine whether the following matrix, A, is invertible, and if it is, find its inverse.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 3 & 2 \end{bmatrix} \quad .$$

Ans.: $A^{-1} =$

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15. (10 points) Determine whether the given system is consistent, and if so, find its general solution, expressed in **vector form**.

$$x_1 - x_2 + x_4 = -4 \quad ,$$

$$x_1 - x_2 + 2x_4 + 2x_5 = -5 \quad ,$$

$$3x_1 - 3x_2 + 2x_4 - 2x_5 = -11 \quad .$$

Ans.:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} =$$

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16. (10 points) Find an elementary matrix E such that $EA = B$, where A and B are as follows:

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 3 & -1 & 0 \\ -1 & 1 & 6 \end{bmatrix} \quad \textit{and} \quad B = \begin{bmatrix} 1 & 2 & -2 \\ 3 & -1 & 0 \\ 0 & 3 & 4 \end{bmatrix} \quad .$$

Ans.: $E =$

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17. (10 points) Find the inverse of the following matrix, or explain why it does not exist.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \\ 3 & 3 & 3 & 6 \end{bmatrix} .$$

Ans.:

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18. (10 points)
Compute the matrix-vector product

$$\begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 0 & -2 & -1 \\ -1 & 1 & 2 & 0 \\ 1 & 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} a+b+d \\ a-d \\ a+c+b \\ a-d \end{bmatrix} \quad ,$$

where a, b, c, and d are real numbers.

Ans.:

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19. (10 points) Find a subset of the following set $\mathcal{S}$ of vectors in R^4 with the same span as $\mathcal{S}$ that is as small as possible.

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} , \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} , \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} , \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \right\} .$$

Ans.:

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20. (10 points) The *reduced row echelon form* of a certain system of linear equations is:

$$\left[\begin{array}{ccccc|c} 1 & -3 & 2 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad .$$

Determine whether this system is consistent, and if so, find its general solution. In addition, write the solution in *vector form*.

Ans.:
