Solutions to Dr. Z.'s Math 250(1), (Fall 2010, RU) REAL Quiz #9 (Dec. 2, 2010)

1. (4 points) A matrix A is given. Find, if possible, an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Otherwise explain why A is not diagonalizable.

$$A = \begin{bmatrix} 1 & 5 \\ -1 & -1 \end{bmatrix} \quad ,$$

Sol. to 1:

$$\det(A - tI_2) = \det \begin{bmatrix} 1-t & 5 \\ -1 & -1-t \end{bmatrix} = (1-t)(-1-t)+5 = (t-1)(t+1)+5 = t^2-1+5 = t^2+4 \quad .$$

The **eigenvalues** are the roots of the quadratic equation $t^2 + 4 = 0$ whose roots are $\pm\sqrt{-4} = \pm 2i$. Since these are complex numbers, it means that the matrix A has no real eigenvalues, and hence it is **impossible** to diagonalize it (over the real numbers, it is possible to do it using complex numbers, but this is not allowed in this course).

Ans. to 1: Impossible

2. (4 points) Use **1** to compute A^4 (No credit for other methods!)

Reminder 1: $5^4 = 625$

Reminder 2 The quickest way to find the inverse of a 2×2 matrix is to use the formula:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad .$$

Sol. to 2: Since it is impossible to diagonalize A (over the real numbers) it is impossible to find A^4 by this method. Of course, it is possible to do it diretly, and it is also possible to do it if you allow D and P to have complex numbers, but you are not expected to know how to do it.

3. (1 point each). True or False. Explain briefly!

(a) Every diagonalizable 4×4 matrix has 4 distinct eigenvalues.

Sol. to 3a): **False**. It is possible that the characteristic polynomial would have multiple roots. For example, the identity matrix I_4 is diagonalizable and it has the eigenvalue $\lambda = 1$ repeated 4 times.

(b) A diagonal matrix is diagonalizable.

Sol. to 3b): **True**. A diagonal matrix is the epitome of a diagonalizable matrix, simply take $P = I_n$.