Solutions to Dr. Z.'s Math 250(2), (Fall 2010, RU) REAL Quiz #7 (Nov. 4, 2010)

1. (5 points) Find a basis for the null space of the matrix

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Sol. of 1: We find the **reduced row echelon form** of the matrix:

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \overset{r_3 - r_1 \to r_3}{\longrightarrow} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix} \overset{r_3 - 2r_2 \to r_3}{\longrightarrow}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \overset{-r_2 \to r_2}{\longrightarrow} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \overset{r_1 - 2r_2 \to r_1}{\longrightarrow} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

In everyday notation the system $R\mathbf{x} = \mathbf{0}$ is

$$x_1 + 2x_3 = 0 \quad ,$$

$$x_2 - x_3 = 0 \quad ,$$

$$0 = 0 \quad .$$

Solving we get:

$$x_1 = -2x_3$$

$$x_2 = x_3$$

$$x_3 = x_3 \quad (free)$$

In **vector notation** this is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

Since there is only one free variable (x_3), the dimension of the null space (alias the nullity of A) is 1 and a basis is:

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\} \quad .$$

2. (5 points, 1 each) Deteremine whether the following statements are True or False. Give a short expalanation (or an example or counter-example) in each case. (**No credit without explanation**).

(a) The subspace $\{\mathbf{0}\}$ is called the null-space.

Sol. of 2a): **False**. It is called the **zero-space**.

(b) If $\mathcal{S}$ is a linearly independent subset and Span $\mathcal{S}$=V, then $\mathcal{S}$ is a generating set for V.

Sol. of 2b): **True**. It sure is. It is enough that Span $\mathcal{S}$=V to make $\mathcal{S}$ a generating set, so the fact that it is linearly independent is irrelevant.

(c) R^3 contains at least three different subspaces.

Sol. of 2c): **True**. 3 is a very gross underestimate! It has infinitely many subspaces.

(d) A vector $\mathbf{v}$ is in $ColA$ if and only if $A\mathbf{x} = \mathbf{v}$ is consistent.

Sol. of 2d): **True**. If a vector $\mathbf{v}$ is in $ColA$ it means that it is a **linear combination** of its columns, that means that it can be expressed in the form $A\mathbf{x}$ for some vector of numbers $\mathbf{x}$ which means that the system of equations $A\mathbf{x} = \mathbf{v}$ has a solution, which means that the system of equations $A\mathbf{x} = \mathbf{v}$ is **consistent**.

(e) The pivot columns of the matrix A (i.e. the columns corresponding to the pivot columns of the reduced-row-echelon form R, but those of A, not of R) always form a basis for its column space.

Sol. of 2e): **True**. By the column correspondence property. The pivot columns of R obviously form a basis for **its own** column space (not to be confused with the column space of A that is (usually) entirely different). By the column correspondence property it follows that the correponding columns in A form a basis for the column space of A.