Solutions to Dr. Z.'s Math 250(1), (Fall 2010, RU) REAL Quiz #6 (Oct. 28, 2010)

1. Compute the determinant of the following matrix by a cofactor expansion along the third row.

$$\begin{bmatrix} 3 & -1 & 1 \\ 4 & -1 & 4 \\ -2 & 1 & 2 \end{bmatrix}$$

Solution of 1:

$$\det \begin{bmatrix} 3 & -1 & 1 \\ 4 & -1 & 4 \\ -2 & 1 & 2 \end{bmatrix} = (-2)\det \begin{bmatrix} -1 & 1 \\ -1 & 4 \end{bmatrix} - (1)\det \begin{bmatrix} 3 & 1 \\ 4 & 4 \end{bmatrix} + (2)\det \begin{bmatrix} 3 & -1 \\ 4 & -1 \end{bmatrix}$$

$$(-2)((-1)(4)-(1)(-1))-(1)((3)(4)-(1)(4))+(2)((3)(-1)-(-1)(4)) = (-2)(-3)-(1)(8)+(2)(1) = 6-8+2 = 0$$

Ans. to 1: 0.

Comment: Most people got it right. Some people messed up the arithmetics.

2. Compute the determinant of the following matrix by using elementary operations.

$$\begin{bmatrix} 3 & 4 & 2 \\ 2 & -1 & 3 \\ -1 & 3 & 1 \end{bmatrix} \quad .$$

Sol. of 2:

$$\begin{bmatrix} 3 & 4 & 2 \\ 2 & -1 & 3 \\ -1 & 3 & 1 \end{bmatrix} \overset{r_1 \leftrightarrow r_3}{\longrightarrow} \begin{bmatrix} -1 & 3 & 1 \\ 2 & -1 & 3 \\ 3 & 4 & 2 \end{bmatrix} \begin{matrix} r_2 + 2r_1 \to r_2 \\ \longrightarrow \\ r_3 + 3r_1 \to r_3 \end{matrix} \begin{bmatrix} -1 & 3 & 1 \\ 0 & 5 & 5 \\ 0 & 13 & 5 \end{bmatrix} \overset{r_3 - (13/5)r_2 \to r_3}{\longrightarrow} \begin{bmatrix} -1 & 3 & 1 \\ 0 & 5 & 5 \\ 0 & 0 & -8 \end{bmatrix} \quad .$$

Now it is an **upper-triangular matrix** and the determinant of this last matrix is simply the product of the diagonal entires: $(-1)(5)(-8) = 40$. **But** we had one swapping operator, so the answer is $(-1)^1 40 = -40$.

Ans. to 2: -40.

Comments: About %60 of the people got it right. Some people made it much too complicated, by not swapping r_1 and r_3 first. Of those people some got the right answer, but some messed up because the numbers got messy. Quite a few people used **illegal** row-operations of the type $dr_i + cr_j \to r_i$, with d a number that is not 1. This is **not** an elementary-row operation! For example, $r_i + 2r_j \to r_i$ is legal as is $r_i - 5r_j \to r_i$, but $2r_i + 3r_j \to r_i$ is **illegal**.