Dr. Z.'s Math 250(1), (Fall 2010, RU) Solutions to the REAL Quiz #3 (Sept. 30, 2010)

1. (4 pts.) Find (i) the $(2,3)$ entry of AC ; (ii) the $(3,2)$ entry of CA ; if the matrices A and C are:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ -1 & 5 & -1 \end{bmatrix} \quad , \quad C = \begin{bmatrix} -1 & 4 & 3 \\ 0 & 6 & -2 \\ -2 & 3 & 0 \end{bmatrix} \quad .$$

Sol. of 1 (i): The $(2,3)$ entry of AC is the **dot product** of Row-2 of A and Column-3 of C:

$$[-1 \quad 0 \quad 3] \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} = (-1)(3) + (0)(-2) + (3)(0) = -3 \quad .$$

Sol. of 1 (ii): The $(3,2)$ entry of CA is the **dot product** of Row-3 of C and Column-2 of A:

$$[-2 \quad 3 \quad 0] \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} = (-2)(2) + (3)(0) + (0)(5) = -4 \quad .$$

Comments: Most people got it right. A few people, by mistake did for part (ii) , the $(3,2)$ entry of AC (make sure to read the question carefully!)

2. (4 pts.) Determine whether the given set is linearly independent

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} , \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} , \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\} \quad .$$

Sol. of 2: We form the matrix whose columns are the given set and find its **rank** by finding its **row-echelon form**.

$$\begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

In order to get the entries below the pivot of the first row (namely entries $(2,1)$ and $(3,1)$) to be zero, we perform the elementary row operations: $r_2 + r_1 \to r_2$ and $r_3 - r_1 \to r_3$, and get

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 3 \\ 0 & 3 & -1 \end{bmatrix}$$

In order to get the entry below the pivot of the second row (namely entry $(3,2)$) to be zero, we perform the elementary row operation $r_3 + 3r_2 \to r_3$ and get:

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 8 \end{bmatrix} \quad .$$

(No need to go further to reduced row-echelon form!). Now that it is in row-echelon form we see that there are **three** pivots (i.e. no all-zeroes-rows) so the rank is 3 and this means that the above set is **linearly independent**.

Comments: Most people did it correctly, **but** quite a few worked harder than necessary, by going all the way to the reduced-row-echelon form. Once you are done with phase I, and realize that the rank is n (3 in our case), you are done, and the set is **linearly independent**.

3. True or False (Explain when appropriate, and if it is false correct the statement to make it true).

(a) (1 pt.) If the only solution of $A\mathbf{x} = \mathbf{0}$ is $\mathbf{0}$, then the rows of A are linearly independent.

Sol of 3(a): False, the corrected statement is when you replace "rows" by "columns".

(b) (1 pt.) There exist non-zero matrices A and B such that $AB = BA$.

Sol of 3(b): True, for example when $A = B$ or when either A or B are the **identity matrix**.