Dr. Z.'s Math 250(2), (Fall 2010, RU) Solutions to REAL Quiz #2 (Sept. 23, 2010)

1. (4 pts.) Determine the value of r, if any, for which the given system of linear equations is **inconsistent**.

$$x_1 - x_2 + 2x_3 = 4$$

$$3x_1 + rx_2 - x_3 = 2 \quad .$$

Sol. of 1: We pretend that r is just a number. The augmented matrix is:

$$\begin{bmatrix} 1 & -1 & 2 & 4 \\ 3 & r & -1 & 2 \end{bmatrix} \quad .$$

Let's try to bring it to **row-echelon form** (no need to go all the way to **reduced row echelon form**). Doing the elementary row operation $r_2 - 3r_1 \to r_2$ we get

$$\begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & r+3 & -7 & -10 \end{bmatrix} \quad .$$

This is row-echelon form. We now look for the scenario of an all-zero-row-except-for-the-very-last-one, and this can never happy. Even if $r = -3$ we still have $0, 0, -7, -10$ so the -7 protects us. So the answer is **never!**. The system is always consistent! (i.e. it is never inconsistent) no matter what r is.

Ans. : No values of r.

2. (4 pts.) Find a subset of the following set $\mathcal{S}$ of vectors in R^3 with the same span as $\mathcal{S}$ that is as small as possible.

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad , \quad \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \quad , \quad \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \right\} \quad .$$

Sol. of 2: By inspection the last vector is the sum of the first two, so we can **kick-out** the third vector getting

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad , \quad \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\} \quad .$$

Now, amongst the two survivors, none is a scalar-multiple of the other, so we need them both. So this is the **answer**.

3. True or False (Explain!, unexplained answered get no credit)

(a) (1 pt.) The number of rows in a matrix equals its rank.

Sol. False. The rank equals the number of non-zero-rows in the **reduced row-echelon form** (also in any row-echelon form).

(b) (1 pt.) If **u** and **v** are any two vectors in R^2, then their span is the whole of R^2.

Sol.: False. If **u** and **v** are parallel (multiples of each other) than they only span a one-dimensional subspace. The corrected statemet is:

If **u** and **v** are any two **non-parallel** vectors in R^2, then their span is the whole of R^2.