1. (4 pts.) Determine the value of $r$, if any, for which the given system of linear equations is inconsistent.

$$ x_1 - x_2 + 2x_3 = 4 $$
$$ 3x_1 + rx_2 - x_3 = 2 $$

**Sol. of 1:** We pretend that $r$ is just a number. The augmented matrix is:

$$ \begin{bmatrix} 1 & -1 & 2 & 4 \\ 3 & r & -1 & 2 \end{bmatrix} $$

Let’s try to bring it to row-echelon form (no need to go all the way to reduced row echelon form). Doing the elementary row operation $r_2 - 3r_1 \rightarrow r_2$ we get

$$ \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & r+3 & -7 & -10 \end{bmatrix} $$

This is row-echelon form. We now look for the scenario of an all-zero-row-except-for-the-very-last-one, and this can never happen. Even if $r = -3$ we still have 0, 0, $-7$, $-10$ so the $-7$ protects us. So the answer is **never**! The system is always consistent! (i.e. it is never inconsistent) no matter what $r$ is.

**Ans.** : No values of $r$.

2. (4 pts.) Find a subset of the following set $S$ of vectors in $\mathbb{R}^3$ with the same span as $S$ that is as small as possible.

$$ S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \right\} $$

**Sol. of 2:** By inspection the last vector is the sum of the first two, so we can **kick-out** the third vector getting

$$ \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\} $$

Now, amongst the two survivors, none is a scalar-multiple of the other, so we need them both. So this is the **answer**.

3. True or False (Explain!, unexplained answered get no credit)

(a) (1 pt.) The number of rows in a matrix equals its rank.

**Sol.** False. The rank equals the number of non-zero-rows in the **reduced row-echelon form** (also in any row-echelon form).
(b) (1 pt.) If \( u \) and \( v \) are any two vectors in \( \mathbb{R}^2 \), then their span is the whole of \( \mathbb{R}^2 \).

**Sol.** False. If \( u \) and \( v \) are parallel (multiples of each other) than they only span a one-dimensional subspace. The corrected statement is:

If \( u \) and \( v \) are any two **non-parallel** vectors in \( \mathbb{R}^2 \), then their span is the whole of \( \mathbb{R}^2 \).