MATH 250, Fall 2010, Sections 1 and 2 (Dr. Z.) , Answers (and some hints and sketches) for Practice Problems for Exam 2

Last Update: Improving the answer of #27 (the basis for the eigenspace corresponding to the eigenvalue $t = 1$ I gave before, produced by Maple, was correct, but it is not the one that you get from doing the rref method.) [Noticed by Christina Malleo]

Previous Updates: Nov. 13, 2010, 1:22pm (correcting a mistake found by Mike Monico in #27)

Nov. 12, 2010, 7:30pm, correcting a typo found by Adrien Perkins (31(c) was F, but of course it is T!) Congratulations, Adrien, for winning $2!

Nov. 11, 2010, 3:00pm, correcting a typo found by Sarita Paul (24(b) was before 16 the correct answer is 32). Congratulations, Sarita, for winning $2!

Disclaimer: Not responsiple for errors. An award of $2 for the first finder of any error. A $1 fine for any false alarm. In the case of proofs, I only gave brief sketches in *parentheses*. The full answer should be fuller!

1. (a)

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{bmatrix} \quad , \quad U = \begin{bmatrix} -1 & 2 & 1 & -1 & 3 \\ 0 & -2 & 1 & 4 & -2 \\ 0 & 0 & -2 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $x_1 = 1 + 4x_4, x_2 = -3 + \frac{9}{4}x_4, x_3 = 2 + \frac{1}{2}x_4, x_4 = x_4, x_5 = 4$

2. (a) 15 (use elemetary row operations to get it to be "almost" A, but with $\mathbf{a}$ replaced by $3\mathbf{a}$, so the value of the determinant 3 times the determinant of A) (b) 200 (First find $\det C$, that is 2, and then use $\det(AC^2AC) = \det(A)\det(C)^2\det(A)\det(C) = \det(A)^2\det(C)^3$).

3. 6.

4. (a) (i)

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) (i)

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. $-\frac{6}{13}$.

6. $3\mathbf{a_1} - 2\mathbf{a_2} + \mathbf{a_3} = \mathbf{0}$.

7. (Check the three properties of a subspace, or express this set as a span of three specific vectors). A basis is:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \\ -2 \\ 1 \end{bmatrix} , \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \\ -1 \end{bmatrix} , \begin{bmatrix} 2 \\ 0 \\ 5 \\ 1 \\ -1 \end{bmatrix} \right\}$$

8. (a) $(\mathbf{v}\mathbf{v}^T)\mathbf{v} = \mathbf{v}(\mathbf{v}^T\mathbf{v})$ (by associativity). But $(\mathbf{v}^T\mathbf{v})$ is a **number** (a 1×1 matrix), so we can move it to the left and get $(\mathbf{v}\mathbf{v}^T)\mathbf{v} = (\mathbf{v}^T\mathbf{v})\mathbf{v}$. But this means that $\mathbf{v}$ is an eigenvector with eignevalue $\mathbf{v}^T\mathbf{v}$.

(b) 1, 0.

9. (a) (Find a vector in the set whose negative is not in the set). (b) (Prove that $\mathbf{0}$ is not a member of this set).

10. (a) -2 (multiplicity 2); 5 (multiplicity 1) . (b) $t = -2$: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$; $t = 5$: $\left\{ \begin{bmatrix} 1 \\ -5 \\ -6 \end{bmatrix} \right\}$.

11. (a) (For each of them check that the three defining properties for a subspace are satisfied.) (b) (Find an element of V and an element of W such that if you add them up, you get something neither in V nor in W).

12.

(a) (Use the fact that $\det(AB) = \det(A)\det(B)$ and $\det(B^{-1}) = 1/\det B$

(b) (Use the fact that $\det(Q^T) = \det(Q)$ and the above.)

13. (A basis for R^3 has **exactly** three elements.)

14. (Use elementary row operations)

15. a)2 b)2 c) 2 d) 1

16. (a) (Come up with two elements whose sum is not a member); (b) (Come up with a member whose double is not a member).

17. (a)F (b)F (c)T (d)T

18. (a) T (b)T (c) T (d) F

19. (First check that the two members of the proposed basis belong to V, then compute the dimension of V, then show that the two members of the proposed basis are linearly independent)

20. (Start with $A\mathbf{v} = \lambda\mathbf{v}$. Multiply A from the left to both sides of the equation, and keep going).

21.

$$(i)\left\{\begin{bmatrix}1\\-1\\2\end{bmatrix}, \begin{bmatrix}1\\-2\\3\end{bmatrix}, \begin{bmatrix}-1\\1\\1\end{bmatrix}\right\}, \quad (ii)\left\{\begin{bmatrix}-2\\1\\-3\\1\end{bmatrix}\right\}.$$

22. $-22x - 6y + 8z$

23. $x_1 = \frac{5}{2}, x_2 = -1, x_3 = \frac{1}{2}$ (but you had to use Cramer).

24. (a)16 (b) 32

25.

$$\left\{\begin{bmatrix}4\\6\\-10\end{bmatrix}, \begin{bmatrix}8\\-12\\20\end{bmatrix}, \begin{bmatrix}1\\0\\-2\end{bmatrix}\right\}$$

26. (R^3, being **three**-dimensional needs at least three elements to generate it.)

27.

$$\lambda = 1(mult.\ 2) : \left\{\begin{bmatrix}0\\-1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\-2\end{bmatrix}\right\}, \quad \lambda = -2(mult.\ 2) : \left\{\begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}\right\}$$

28. (Compute the characteristic polynomial, and use the quadratic equation to solve it).

29. (a) F (m should be n) (b) T (c) T (d) T

30. (If $A\mathbf{v} = \lambda\mathbf{v}$, then mulitply both sides by A getting $A^2\mathbf{v} = \lambda\mathbf{Av} = \lambda\lambda\mathbf{v} = \lambda^2\mathbf{v}$. Keep going, getting: $A^4\mathbf{v} = \lambda^4\mathbf{v}$ and get $O\mathbf{v} = \lambda^4\mathbf{v}$ so $\lambda^4 = 0$)

31. (a) F (b) T (c) T (d) T
[Note: thanks to Adrien Perkins for correcting the ans. to (c)]

32. yes, inverse is:

$$\begin{bmatrix} -2 & 1 & 1 \\ -1 & 0 & 1 \\ 3 & -1 & -1 \end{bmatrix} \quad .$$