Solutions to the Attendance Quiz for Sept. 30, 2010

1. Determine whether the matrix is invertible. If so find, its inverse.

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 5 \\ 1 & 3 & 1 \end{bmatrix}$$

Sol. of 1: First we try to bring the matrix into **reduced row-echelon form** (of course, if it is not invertible, we can stop after phase I, declaring that it is not invertible, otherwise we continue to the end).

First apply the elementary row operations $r_2 - 2r_1 \to r_2$ and $r_3 - r_1 \to r_3$ to make the $(2,1)$ and the the $(3,1)$ entries 0, getting:

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \quad .$$

This is already in row-echelon form, so we know that the matrix is **invertible** and we must go on to the reduced-row-echelon form. Now we go **bottom-up**.

First me have to make all pivots 1, so we do $-r_2 \to r_2$, $-r_3 \to r_3$ getting:

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad .$$

Doing $r_2 + r_3 \to r_2$ and $r_1 - 2r_3 \to r_1$ yields

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ,$$

and doing $r_1 - 3r_2 \to r_1$ gives:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad .$$

This is the reduced row-echelon-form and since it is the **identity matrix** I_3 the matrix is indeed invertible (of course we already knew that!, from phase I).

Now let's collect, **in order** all the elemenary row operations that we performed:

(i) $r_2 - 2r_1 \to r_2$

(ii) $r_3 - r_1 \to r_3$

(iii) $-r_2 \to r_2$

(iv) $-r_3 \to r_3$

(v) $r_2 + r_3 \to r_2$

(vi) $r_1 - 2r_3 \to r_1$

(vii) $r_1 - 3r_2 \to r_1$

Applying (i) and (ii) to the **identity matrix** gives

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Applying (iii) and (iv) gives

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

Applying (v) and (vi) yields

$$\begin{bmatrix} -1 & 0 & 2 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Applying (vii) yields:

$$\begin{bmatrix} -10 & 3 & 5 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Ans. to 1: The matrix is invertible and its inverse is the matrix

$$\begin{bmatrix} -10 & 3 & 5 \\ 3 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Comments: Only about %40 of the people got it completely, but most people did it the right way.

2. Use the algorithm for computing $A^{-1}B$, where

$$A = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$$

Sol. of 2: We bring the matrix A to reduced-row-echelon form.

First we do $r_2 + 2r_1 \to r_2$ to get

$$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$$

This is already in row-echelon-form, so A is indeed invertible. Continuing to phase II we first do $-r_1 \to r_1$, getting:

$$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

followed by $r_1 + 2r_2 \to r_1$ yielding

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad .$$

So, as expected, the reduced-row-echelon form of A is the identity matrix I_2.

Let's record the elementary-row-operations that got us here, in order

(i) $r_2 + 2r_1 \to r_2$

(ii) $-r_1 \to r_1$

(iii) $r_1 + 2r_2 \to r_1$

Now apply these operations, **in that order** to the matrix B.

Applying operation (i) (namley $r_2 + 2r_1 \to r_2$) gives

$$\begin{bmatrix} 4 & -1 \\ 9 & 0 \end{bmatrix}$$

Applying operation (ii) (namley $-r_1 \to r_1$) yields

$$\begin{bmatrix} -4 & 1 \\ 9 & 0 \end{bmatrix} \quad ,$$

and finally, applying operation (iii) (namely $r_1 + 2r_2 \to r_1$) yields

$$\begin{bmatrix} 14 & 1 \\ 9 & 0 \end{bmatrix} \quad .$$

Ans. to 2:

$$A^{-1}B = \begin{bmatrix} 14 & 1 \\ 9 & 0 \end{bmatrix} \quad .$$

Comments: About %60 of the people got the correct answer. Another %20 used the right method, but got the wrong answer. Quite a few people did it the long way. They first found A^{-1} like in problem 1, and then they did the matrix multiplication $A^{-1}B$. This is a **waste of time**! Of course for a 2×2 matrix it is not that difficult, but try to do it for a 5×5 matrix. Multiplying two five-by-five matrices takes for ever, so **please** use the Gaussian elimination method, remember it is the same effort as finding the inverse, A^{-1} but instead of starting with the identity matrix I_n, you start with the matrix B.