If

$$A^{-1} = \begin{bmatrix} -2 & 4 \\ 5 & 7 \end{bmatrix} \quad , \quad B^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \quad ,$$

Compute the following expressions

1.
$$(AB)^{-1}$$

Sol. of 1:

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} (1)(-2) + (-1)(5) & (1)(4) + (-1)(7) \\ (2)(-2) + (3)(5) & (2)(4) + (3)(7) \end{bmatrix} = \begin{bmatrix} -7 & -3 \\ 11 & 29 \end{bmatrix} .$$

Comment: About %70 of the students got it perfectly. Another %10 messed up the calculations, but did it the right way. About %10 of the people did $A^{-1}B^{-1}$ instead of the correct $B^{-1}A^{-1}$.

2.
$$(BA)^{-1}$$

Sol of 2.

$$(BA)^{-1} = A^{-1}B^{-1} = \begin{bmatrix} -2 & 4 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} (-2)(1) + (4)(2) & (-2)(-1) + (4)(3) \\ (5)(1) + (7)(2) & (5)(-1) + (7)(3) \end{bmatrix} = \begin{bmatrix} 6 & 14 \\ 19 & 16 \end{bmatrix} .$$

Comment: About %70 of the students got it perfectly. Another %10 messed up the calculations, but did it the right way. About %10 of the people did $B^{-1}A^{-1}$ instead of the correct $A^{-1}B^{-1}$.

3.
$$(A^T)^{-1}$$

Sol. of 3:

$$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 5\\ 4 & 7 \end{bmatrix}$$
.

Comment: About %75 of the students got it perfectly. Some people got confused how to find the transpose of a matrix.

4.
$$(AB^T)^{-1}$$

Sol. of 4:

$$(AB^T)^{-1} = (B^T)^{-1}A^{-1} = (B^{-1})^TA^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} (1)(-2) + (2)(5) & (1)(4) + (2)(7) \\ (-1)(-2) + (3)(5) & (-1)(4) + (3)(7) \end{bmatrix} = \begin{bmatrix} 8 & 18 \\ 17 & 17 \end{bmatrix}$$

Comment: Only about %60 of the students got it perfectly. Some people ran out of time.