Let

$$
A=\left[\begin{array}{cc}
-2 & 4 \\
5 & 7
\end{array}\right] \quad, \quad B=\left[\begin{array}{cc}
1 & -1 \\
2 & 3
\end{array}\right] \quad, \quad C=\left[\begin{array}{ccc}
1 & -1 & 3 \\
2 & 3 & 0
\end{array}\right]
$$

Compute the following expressions, or give a reason why the expression is nonsense.

1. $A B$

Sol. of 1:

$$
\begin{gathered}
A B=\left[\begin{array}{cc}
-2 & 4 \\
5 & 7
\end{array}\right]\left[\begin{array}{cc}
1 & -1 \\
2 & 3
\end{array}\right] \\
=\left[\begin{array}{cc}
(-2) \cdot(1)+(4) \cdot(2) & (-2) \cdot(-1)+(4) \cdot(3) \\
(5) \cdot(1)+(7) \cdot(2) & (5) \cdot(-1)+(7) \cdot(3)
\end{array}\right]=\left[\begin{array}{cc}
-2+8 & 2+12 \\
5+14 & -5+21
\end{array}\right]=\left[\begin{array}{cc}
6 & 14 \\
19 & 16
\end{array}\right] .
\end{gathered}
$$

Comment: About $\% 85$ of the people got it right. Most of the other people did it the right way, but messed up with the arithmetics.
2. $B A$

Sol. of 2 :

$$
\begin{gathered}
B A= \\
{\left[\begin{array}{cc}
1 & -1 \\
2 & 3
\end{array}\right]\left[\begin{array}{cc}
-2 & 4 \\
5 & 7
\end{array}\right]}
\end{gathered}
$$

$$
=\left[\begin{array}{cc}
(1) \cdot(-2)+(-1) \cdot(5) & (1) \cdot(4)+(-1) \cdot(7) \\
(2) \cdot(-2)+(3) \cdot(5) & (2) \cdot(4)+(3) \cdot(7)
\end{array}\right]=\left[\begin{array}{cc}
-2-5 & 4-7 \\
-4+15 & 8+21
\end{array}\right]=\left[\begin{array}{cc}
-7 & -3 \\
11 & 29
\end{array}\right]
$$

Comment: About $\% 85$ of the people got it right. Most of the other people did it the right way, but messed up with the arithmetics.
3. $A C^{T}$

Sol. of 3: $A$ is a $2 \times 2$ matrix and $C^{T}$ is a $3 \times 2$ matrix.
You can't multiply them, since the number of columns of $A(2)$ is different than the number of rows (3) of $C^{T}$.

Comment: About $\% 75$ of the people got it right. Some people got confused and did $(A C)^{T} . A C^{T}$ means you first take the transpose of $C$, and then multiply $A$ by $C^{T}$.
4. $B C$

Sol. of 4:

$$
B C=\left[\begin{array}{cc}
1 & -1 \\
2 & 3
\end{array}\right]\left[\begin{array}{ccc}
1 & -1 & 3 \\
2 & 3 & 0
\end{array}\right]
$$

$$
=\left[\begin{array}{ccc}
(1) \cdot(1)+(-1) \cdot(2) & (1) \cdot(-11)+(-1) \cdot(3) & (1) \cdot(3)+(-1) \cdot(0) \\
(2) \cdot(1)+(3) \cdot(2) & (2) \cdot(-1)+(3) \cdot(2) & (3) \cdot(3)+(3) \cdot(0)
\end{array}\right]=\left[\begin{array}{ccc}
1-2 & -1-3 & 3+0 \\
2+6 & -2+9 & 6+0
\end{array}\right]=\left[\begin{array}{ccc}
-1 & -4 & 3 \\
8 & 7 & 6
\end{array}\right]
$$

Comment: About $\% 70$ of the people got it right. Some of the other people did it the right way, but messed up with the arithmetics, and some ran out of time.

