Solutions to the Attendance Quiz for Nov. 29, 2010

1. Consider the vectors $u$ and $v$:

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v = \begin{bmatrix} -11 \\ 4 \\ 1 \end{bmatrix}$$

(a) Prove that $u$ and $v$ are orthogonal to each other.

Sol. of 1a):

$$u \cdot v = (1)(-11) + (2)(4) + (3)(1) = -11 + 8 + 3 = 0.$$  
Since the dot-product of $u$ and $v$ is 0, it follows that they are orthogonal.

(b) Compute the quantities $||u||^2$, $||v||^2$ and $||u + v||^2$. Use your results to prove the Pythagorean theorem.

Sol. of 1b): First, let’s compute $u + v$:

$$u + v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -11 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ 6 \\ 4 \end{bmatrix}.$$  
Now:

$$||u||^2 = (1)^2 + (2)^2 + (3)^2 = 1 + 4 + 9 = 14$$

$$||v||^2 = (-11)^2 + (4)^2 + (1)^2 = 121 + 16 + 1 = 138$$

$$||u + v||^2 = (-10)^2 + (6)^2 + (4)^2 = 100 + 36 + 16 = 152.$$  
Since $14 + 138 = 152$, Pythagoras was proven right! (at least in this case).

2. Suppose that $u$, $v$, $w$ are vectors in $\mathbb{R}^n$ such that $u \cdot v = 2$, $u \cdot w = 3$, and $v \cdot w = -2$. Compute $(u + w) \cdot v$.

Sol. of 2: Using the distributive property of the dot-product, we have

$$(u + w) \cdot v = u \cdot v + w \cdot v$$

By the commutative property we have that $w \cdot v = v \cdot w$, so we have

$$(u + w) \cdot v = u \cdot v + w \cdot v = u \cdot v + v \cdot w.$$  
Now use the data of the problem to get that this equals:

$$2 + (-2) = 0.$$  
Ans. to 2: 0.