Solutions to the Attendance Quiz for Nov. 1, 2010

1. Determine the dimension of the following subspace. Explain what you are doing!

$$\left\{ \begin{bmatrix} 2s \\ -s+4t \\ s-3t \end{bmatrix} \in R^3 \; : \; s \quad and \quad t \quad are \quad scalars \right\} \quad .$$

Sol. to 1: Separating the s and t out, the same set can be written as

$$\left\{ s \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 4 \\ -3 \end{bmatrix} \; : \; s \quad and \quad t \quad are \quad scalars \right\} \quad .$$

So the set

$$\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \quad , \quad \begin{bmatrix} 0 \\ 4 \\ -3 \end{bmatrix} \right\} \quad .$$

is a **generating set**. Since we only have **two** vectors here, and, by inspection they are not multiples of each other, we have that they are **linearly independent**. So we have a **basis**. Since this basis has **two** members, the **dimension** is 2.

Ans. to 1: The dimension of this subspace is 2.

Comment: About %75 of the people got it right.

2. Find a basis for *Row A*, if

$$A = \begin{bmatrix} 1 & -2 & 1 & -1 & -2 \\ 3 & -6 & 3 & -3 & -6 \\ 2 & -4 & 1 & 1 & 1 \end{bmatrix} \quad .$$

Sol. of 2: We use **Gaussin elimination**, but it is enough only to get it to row-echelon-form (no need to go all the way).

$$A = \begin{bmatrix} 1 & -2 & 1 & -1 & -2 \\ 3 & -6 & 3 & -3 & -6 \\ 2 & -4 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} r_2 - 3r_1 \to r_2 \\ \longrightarrow \\ r_3 - 2r_1 \to r_3 \end{matrix} \begin{bmatrix} 1 & -2 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & 5 \end{bmatrix} \begin{matrix} r_2 \leftrightarrow r_3 \\ \longrightarrow \end{matrix} \begin{bmatrix} 1 & -2 & 1 & -1 & -2 \\ 0 & 0 & -1 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad .$$

Now we pick the **non-zero** rows, and that's a **basis** for the row-space of *Row A*.

Ans. to 2: A basis for *Row A* is:

$$\{ [1 \quad -2 \quad 1 \quad -1 \quad -2] \quad , \quad [0 \quad 0 \quad -1 \quad 3 \quad 5] \} \quad .$$

I am happy with this answer, but if you insist that all vectors should be **column vectors**, then the proper answer is the transpose:

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ -1 \\ -2 \end{bmatrix} \quad , \quad \begin{bmatrix} 0 \\ 0 \\ -1 \\ 3 \\ 5 \end{bmatrix} \right\} \quad .$$

Comment: About %50 of the people in section 1 got it right (I ran out of time, so I had to go very fast on how to do this kind of problem). In section 2 there was not enough time, so only about %25 of the people got it.

Warning: Unlike *Col A* where the row-echelon form is only a stepping-stone for finding who are the pivot columns, but for the actual column-space you look at the **corresponding** columns in the **original** matrix A, for *Row A* you look directly at the **not-all-zero** rows of R, and take them as a basis for the *Row A*. It is a **big mistake** to do row-correspondence and go back to the corresponding rows of A, you would get the wrong answer!. In other words, for *Col A* you **must** look at the original A, and it is a **mistake** to take the pivot columns of R, but for *Row A* you **must** (only!) look at R, and never look back at the original matrix A, and it is a big **mistake**, leading (usually) to the wrong answer, if you do go back. So watch out!