Solutions to the Attendance Quiz for Nov. 1, 2010

1. Determine the dimension of the following subspace. Explain what you are doing!

\[ \begin{cases} 
2s \\
-s + 4t \\
-3s + 3t 
\end{cases} \in \mathbb{R}^3 : s \text{ and } t \text{ are scalars} \]

**Sol. to 1:** Separating the \( s \) and \( t \) out, the same set can be written as

\[ \begin{cases} 
2s - 1 \\
-3t 
\end{cases} + t \begin{cases} 
0 \\
-3 
\end{cases} : s \text{ and } t \text{ are scalars} \]

So the set

\[ \begin{cases} 
2 \\
-1 \\
1
\end{cases}, \quad \begin{cases} 
0 \\
4 \\
-3
\end{cases} \]

is a generating set. Since we only have two vectors here, and, by inspection they are not multiples of each other, we have that they are linearly independent. So we have a basis. Since this basis has two members, the dimension is 2.

**Ans. to 1:** The dimension of this subspace is 2.

**Comment:** About 75% of the people got it right.

2. Find a basis for Row \( A \), if

\[ A = \begin{bmatrix} 
1 & -2 & 1 & -1 & -2 \\
3 & -6 & 3 & -3 & -6 \\
2 & -4 & 1 & 1 & 1 
\end{bmatrix} \]

**Sol. of 2:** We use Gaussian elimination, but it is enough only to get it to row-echelon-form (no need to go all the way).

\[ A = \begin{bmatrix} 
1 & -2 & 1 & -1 & -2 \\
3 & -6 & 3 & -3 & -6 \\
2 & -4 & 1 & 1 & 1 
\end{bmatrix} [r_2 - 3r_1 \rightarrow r_2, r_3 - 2r_1 \rightarrow r_3] \begin{bmatrix} 
1 & -2 & 1 & -1 & -2 \\
0 & 0 & 0 & 0 & 0 \\
0 & -1 & 3 & 5 
\end{bmatrix} [r_2 \leftrightarrow r_3] \begin{bmatrix} 
1 & -2 & 1 & -1 & -2 \\
0 & 0 & -1 & 3 & 5 \\
0 & 0 & 0 & 0 & 0 
\end{bmatrix} \]

Now we pick the non-zero rows, and that’s a basis for the row-space of Row \( A \).

**Ans. to 2:** A basis for Row \( A \) is:

\[ \begin{cases} 
[1 & -2 & 1 & -1 & -2] \\
[0 & 0 & -1 & 3 & 5] 
\end{cases} \]
I am happy with this answer, but if you insist that all vectors should be column vectors, then the proper answer is the transpose:

\[
\begin{bmatrix}
1 \\
-2 \\
1 \\
-1 \\
-2
\end{bmatrix}, \quad
\begin{bmatrix}
0 \\
0 \\
-1 \\
3 \\
5
\end{bmatrix}
\]

**Comment:** About 50% of the people in section 1 got it right (I ran out of time, so I had to go very fast on how to do this kind of problem). In section 2 there was not enough time, so only about 25% of the people got it.

**Warning:** Unlike Col A where the row-echelon form is only a stepping-stone for finding who are the pivot columns, but for the actual column-space you look at the corresponding columns in the original matrix A, for Row A you look directly at the not-all-zero rows of R, and take them as a basis for the Row A. It is a big mistake to do row-correspondence and go back to the corresponding rows of A, you would get the wrong answer!. In other words, for Col A you must look at the original A, and it is a mistake to take the pivot columns of R, but for Row A you must (only!) look at R, and never look back at the original matrix A, and it is a big mistake, leading (usually) to the wrong answer, if you do go back. So watch out!