Solutins to the Attendance Quiz for Dec. 9, 2010

1. Find the equation of the least-squares line for the given data:

$$(1,2),(2,4),(3,7) \quad .$$

Sol. of 1:

$$C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \quad , \quad \mathbf{y} = \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \quad .$$

$$C^T C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix} \quad .$$

$$(C^T C)^{-1} = \frac{1}{(3)(14)-(6)(6)} \begin{bmatrix} 14 & -6 \\ -6 & 3 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 14 & -6 \\ -6 & 3 \end{bmatrix}$$

$$C^T \mathbf{y} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} (1)(2)+(1)(4)+(1)(7) \\ (1)(2)+(2)(4)+(3)(7) \end{bmatrix} = \begin{bmatrix} 13 \\ 31 \end{bmatrix} \quad .$$

Finally:

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = (C^T C)^{-1} C^T \mathbf{y} = \frac{1}{6} \begin{bmatrix} 14 & -6 \\ -6 & 3 \end{bmatrix} \begin{bmatrix} 13 \\ 31 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} (14)(13)+(-6)(31) \\ (-6)(13)+(3)(31) \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 182-186 \\ -78+93 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} -4 \\ 15 \end{bmatrix} = \begin{bmatrix} -\frac{4}{6} \\ \frac{15}{6} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} \\ \frac{5}{2} \end{bmatrix} \quad .$$

So $a_0 = -\frac{2}{3}$ and $a_1 = \frac{5}{2}$.

Ans.: The least-squares line is: $y = -\frac{2}{3} + \frac{5}{2}x$.