NAME: (print!) _______________________________________

Section: ____ **E-Mail address:** ____________________

MATH 250 (1), Dr. Z. , Exam 2, Mon., Nov. 15, 2010, 8:40-10:00am, SEC 202

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.
Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

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tot.: (out of 100)

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1. (a) (6 points) Find an LU decomposition of the matrix

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 3 \\ -1 & 2 & 0 & -2 & -2 \\ 2 & -1 & 7 & -1 & 1 \end{bmatrix}$$

(b) (4 points) Use the results of part (a) to solve the system

$$\begin{aligned} x_1 - x_2 + 2x_3 + x_4 + 3x_5 &= -4 \\ -x_1 + 2x_2 \qquad - 2x_4 - 2x_5 &= 9 \\ 2x_1 - x_2 + 7x_3 - x_4 + x_5 &= -2 \end{aligned}$$

2. (10 points) Compute the determinant by using elementary row operations (no credit for other methods)

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & -1 & -1 & 4 \\ -4 & 5 & -10 & -6 \\ 3 & -2 & 10 & -1 \end{bmatrix}$$

3. (10 points) For what values of c is the given matrix **not** invertible.

$$\begin{bmatrix} -1 & 1 & 1 \\ 3 & -2 & -c \\ 0 & c & -10 \end{bmatrix}$$

4. Explain why the following set is a subspace of R^4 and find a basis for it.

$$\left\{ \begin{bmatrix} r+s+2t \\ r-s \\ 3r+2s+5t \\ -2r+3s+t \end{bmatrix} \in R^4 : r, s, \ and \ t \ are \ scalars \right\}$$

5. Explain why the following sets in R^3 are **not** subspaces

(a) (5 points)

$$\left\{ \begin{bmatrix} r \\ 2r \\ 3r \end{bmatrix} \in R^3 : r \geq 0 \right\}$$

(b)

$$\left\{ \begin{bmatrix} 1+r \\ 2+r \\ 3+r \end{bmatrix} \in R^3 : r\, is\ a\ scalar \right\}$$

6. Let

$$V = \left\{ \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \in R^2 : \nu_1 = 0 \right\}$$

$$W = \left\{ \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \in R^2 : \nu_2 = 0 \right\}$$

(a) (5 points) Prove (using the definition of subspace) that V is a subspace of R^2 and that W is a subspace of R^2.

(b) Show that $V \cup W$ is **not** a subspace of R^2.

7. (10 points) Explain why

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ -11 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ -11 \\ 13 \\ 22 \end{bmatrix} \right\}$$

is not a basis for R^4.

8. (10 points, 2.5 points each) Determine the dimensions of (a) *Col* A (b) *Null* A (c) *Row* A and (d) *Null* A^T, if

$$A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & 1 & -2 \\ -1 & 0 & 4 \end{bmatrix}$$

9. (10 ponts. 2.5 each) Classify each statement as true or false and give a brief justification of your answer.

(a) If $A\mathbf{x} = \mathbf{0}$ has a unique solution than the nullspace of A is empty.

(b) If $\mathbf{u}$ and $\mathbf{v}$ belongs to a subspace W of R^n then $5\mathbf{u} + 11\mathbf{v}$ also belongs to W.

(c) A square matrix is invertible if and only of $\det A = 0$.

(d) If A is a 10×13 matrix, then the nullspace of A is not $\{\mathbf{0}\}$.

10. (10 points) Prove that if λ is an eigenvalue of the matrix A, then λ^2 is an eigenvalue of the matrix A^2.