NAME: (print!) ____

Section: ____ **E-Mail address:** ____________________

MATH 250 (1), Dr. Z. , Exam 1, Thurs., Oct. 14, 2010, 8:40-10:00am, SEC 202

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.
Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

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tot.: (out of 100)

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1. (10 pts. altogether) (a) (7 pts) What is the rank of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 1 \end{bmatrix}$$

(b) (3 points) Using part (a) find the nullity of A.

2. (10 pts.) Let

$$\mathcal{S} = \{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad , \quad \begin{bmatrix} -5 \\ -10 \end{bmatrix} \},$$

determine whether the set $\mathcal{S}$ is linearly independent or linearly dependent. In case it is linearly dependent, write the zero vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ explicitly as a non-trivial linear combination of the vectors in $\mathcal{S}$.

3. (10 pts altogether) Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad , \quad C = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Calculate the following matrix products, if they are defined, or expained why they don't make sense.

(a) (5 points) AB

(b) (3 points) AB^T

(c) (2 points) C^2

4. (10 pts.) For the matrix

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

compute the matrix A^8.

5. (10 pts.) For the following matrix A finds its **reduced-row-echelon form**, R, and find an invertible matrix P such that $PA = R$.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

6. (10 pts. altogether) In each case below, give an $m \times n$ matrix R in *reduced row echelon form* satisfying the given condition, or explain why it is impossible to do so.

(a)(4 pts) $m = 2$, $n = 3$ and the equation $R\mathbf{x} = \mathbf{c}$ has a solution for all $\mathbf{c}$.

(b) (4 pts) $m = 2$, $n = 2$ and the equation $R\mathbf{x} = \mathbf{c}$ has a unique solution for all $\mathbf{c}$.

(c) (2 pts) $m = 3$, $n = 3$ and the equation $R\mathbf{x} = \mathbf{0}$ has no solution.

7. (10 pts.) **Without first computing** A^{-1}, find $A^{-1}B$, if

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

8. (10 pts. altogether , 2 each) **True** or **False**? Give a short explanation!
(a) For any $n \times n$ matrices A and B, if $AB = I_n$, then $BA = I_n$.

(b) If A and B are invertible 2×2 matrices, then so is $A + B$.

(c) The sum of *any* two $m \times n$ matrices is always defined.

(d) The product of *any* two 4×9 matrices is never well-defined.

(e) The equation $A\mathbf{x} = \mathbf{b}$ is consistent if and only of $\mathbf{b}$ is a linear combination of the rows of A.

9. (10 pts.) Let $\mathbf{u}$ be a solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{v}$ be a solution of $A\mathbf{x} = \mathbf{0}$, where A is an $m \times n$ matrix and $\mathbf{b}$ is a vector in R^m. Show that $\mathbf{u} + \mathbf{v}$ is a solution of $A\mathbf{x} = \mathbf{b}$.

10. (10 pts. altogether, 5 each) What does it mean to say that the vectors $\mathbf{u}_1, \ldots, \mathbf{u}_k$ in R^n are *linearly independent*? Give the precise definition in one or more sentences.

(b) What is meant by the *span* of a set of vectors $\mathcal{S} = \{\mathbf{u}_1, \ldots, \mathbf{u}_k\}$? Give the precise definition in one or more sentences.