Solutions to Real Quiz # 9 for Dr. Z.'s MathHistory

1. (2 points) What is the name of the French mathematician who did fundamental work in Number Theory independently, and at the same time, as Gauss, and what is the name of the innovative Geomery textbook that he wrote, that broke away from Euclid?

Ans. Adri en Marie Legendre; *Elements de gémoétrie.*

2. (2 points) Who was the most original pupil of Gaspard Monge, and what is the name of the book that he wrote? What does it contain?

Ans. Victor Poncelet; *Trairé des propiétes projectives des figures.*

3. (1 point) What was the position of Evariste Galois's father?

Ans. Small town mayor.

4. (1 point) What is the name of the famous French author that shared Cauchy's conservative political views, as well his capacity for an infinite amount of work?

Ans. Balzac.

5. (4 points total) (a) (1 point) What is

$$\int_{-\infty}^{\infty} e^{-x^2/2}\,dx \quad ?$$

Ans. $\sqrt{2\pi}$

(b) (3 points) Prove it!

Sol. Let

$$c := \int_{-\infty}^{\infty} e^{-y^2/2}\,dy \quad .$$

Hence

$$c^2 = \left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2}\,dy\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dx\,dy \quad .$$

Now we have a **double integral** that, conceptually is more diffucult than a single integral, but surprisingly, it is **easier**, thanks to **polar coordinates**.

Moving to polar coordinates $dx\,dy$ becomes $r\,dr\,d\theta$, and x^2+y^2 becomes r^2. Also, in the polar language, the region of integration becomes $0 \le r < \infty$ and $0 \le \theta < 2\pi$. Hence

$$c^2 = \int_0^{\infty}\int_0^{2\pi} re^{-r^2/2}\,d\theta\;dr \quad .$$

$$= \int_0^\infty re^{-r^2/2} \left(\int_0^{2\pi} d\theta \right) \; dr \; = \; \int_0^\infty re^{-r^2/2}(2\pi) \; dr \quad = \; 2\pi \int_0^\infty re^{-r^2/2} \; dr \quad .$$

But $\frac{d}{dr}(-e^{-r^2/2}) = re^{-r^2/2}$, hence $\int re^{-r^2/2} = -e^{-r^2/2} + C$, and

$$\int_0^\infty re^{-r^2/2} \, d\theta \; dr = -e^{-r^2/2}\Big|_0^\infty = -e^{-\infty} - -e^0 = 1 \quad .$$

Going back above, we get

$$c^2 = 2\pi \quad .$$

So $c = \sqrt{2\pi}$. QED.