

Solutions to Real Quiz # 8 for Dr. Z.'s MathHistory

1. (2 points) What is Laplace's partial differential equation? Who derived it before Laplace?

Ans.

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

Euler.

2. (2 points) What is the name of the city where Carl Friedrich Gauss was born? What was the occupation of his father?

Ans. Brunswick (like our city, New Brunswick!) (in German: Braunschweig). Day Laborer.

3. (2 point) Can the sides of a regular polygon of 17 sides be constructed with compass and rule alone? Who proved (or disproved) it?

Ans. Yes. Gauss **proved** it.

4. (4 points altogether)

Consider the set

$$G = \{0, 1, 2, 3, 4, 5\}$$

where the "multiplication", let's call it $'*$ ', is addition modulo 6, for example: $2'*3=5$, $4'*5=3$.

a (1 point) Show that it is a group.

Sol. of a: If you add up two integers, modulo 6 between 0 and 5 inclusive, you get another one like this, obviously. The identity element is 0, and the inverse of 0 is 0, the inverse of 1 is 5, the inverse of 2 is 4, etc. It is also associative, since addition (even modulo 6) is associative.

b (1 point) Is the subset $\{1, 3, 5\}$ of G a subgroup of G ? Explain!

Sol. of b: No! For example $1'*1 = 2$, and 2 is not a member of this subset.

c (1 point) Show that $H = \{0, 2, 4\}$ a subgroup of G .

Sol. of c: Even plus Even is always even (also modulo 6) and the inverse of 2 is 4 and vice versa, so every element has an inverse. Of course it is associative.

d (1 point) Find the coset decomposition of G with respect to H .

Sol. of d: Take an element of G not in H , for example 1:

$$1' * H = \{1, 3, 5\} \quad ,$$

and now there is nothing left, hence a coset decomposition is

$$G = H \cup 1' *' H \quad .$$