

Solutions to the Real Quiz # 1 for Dr. Z.'s MathHistory

1. (1 points) What are the names of the two papyri that gave us the most information about the math of ancient Egypt?

Ans. to 1: The Rhind papyrus and the Moscow papyrus.

2. (2 points) What was the Egyptian formula for the area of a circle with diameter d ? What was the implied value of π ?

Ans to 2: If d is the diameter, then the area is $(d - \frac{d}{9})^2$. The implied value of π is $(\frac{16}{9})^2 = \frac{256}{81} = 3.16\dots$

3. (2 points) What was the Egyptian formula for the volume of the frustum of a square pyramid for which a and b are the lengths of the sides of the squares, and h is the height?

Ans. to 3:

$$\frac{h}{3}(a^2 + ab + b^2) \quad .$$

Comment: The second part of the question was “what is the implied value of π ?”. I asked it by mistake. This formula has nothing to do with π . I didn’t take any points off for people who gave the same answer $3.16\dots$ as the previous question, but the ideal answer would have been: ”nothing to do with π ”.

4. (2 points) Convert one hundred and one (base ten) to base thirteen. Use the following abbreviations for the digits that represent our ten, eleven, and twelve

ten=A , eleven=B, twelve=C

Sol. to 4: The largest power of 13 smaller than 101 is $13^1 = 13$. Dividing by 13 gives

$$101 = 7 \cdot 13 + 10$$

But 10 is called A, so we get

Ans. to 4 : $7A_{13}$.

5. (3 points) Find the two smallest positive integers n , that have the property that

- If you divide n by 3 you get remainder 2 .
- If you divide n by 4 you get remainder 3 .

Sol. to 5:The ‘stupid’ way is to compile a table of the mapping

$$x \rightarrow (x \pmod{3}, x \pmod{4}) \quad ,$$

for x from 0 to 11. This is correct, but a little time-consuming, and the answer is $n = 11$.

A cleverer way is to note that $n \pmod{3} = -1$, and $n \pmod{4} = -1$. By luck they are the same, so $n = -1$ is an obvious answer. Alas, it is not positive. But if you add to it $3 \cdot 4 = 12$, you get that the smallest positive answer is $n = -1 + 12 = 11$.

To get the second-most positive answer, you add 12 one more time, getting $11 + 12 = 23$.

Ans. to 5: The smallest positive answer is $n = 11$, and the second-smallest is $n = 23$.