**Solutions to Attendance Quiz # 13 for Dr. Z.'s MathHistory for Lecture 13**

**NAME:** (print!) ______________________________

**E-MAIL ADDRESS:** (print!) ______________________________

**1.** Prove that

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad .$$

**Sol.**

We know from calculus that

$$\arctan x = \int_0^x \frac{1}{1+t^2} \quad .$$

The famous infinite geometric series formula (valid for $-1 < z < 1$) is

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \quad .$$

Replacing $z$ by $-t^2$ we get

$$\frac{1}{1+t^2} = \sum_{n=0}^{\infty} (-t^2)^n = \sum_{n=0}^{\infty} (-1)^n t^{2n} \quad .$$

Hence

$$\arctan x = \int_0^x \frac{1}{1+t^2} = \int_0^x \sum_{n=0}^{\infty} (-1)^n t^{2n} = \sum_{n=0}^{\infty} (-1)^n \int_0^x t^{2n} = \sum_{n=0}^{\infty} (-1)^n \left( \frac{t^{2n+1}}{2n+1} \Big|_0^x \right) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad .$$