**NAME: (print!)** ______________________________

**E-Mail address:** ______________________________

**MATH 437 Exam II for Dr. Z.'s, Fall 2021, Dec. 6, 2021, 3:00-4:20pm, (on-line)**

**No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).**
**Show your work! An answer without showing your work will get you zero points.**

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

11. (out of 10)

12. (out of 10)

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total: (out of 120)

**1.** (10 points) Give **two** proofs of the Pythagorean theorem.

**2.** (10 points) Prove that $\sqrt[7]{3}$ is irrational.

**3.** (10 points total)
(a) (5 points) Construct the Pascal triangle mod 2 Fractal using the first 8 rows (i.e, the row for $n = 0$ through row for $n = 7$). Highlight the middle 0 section, and show that the remaining part consists of three identical triangles with 4 rows,

(b) (5 points) Define the Feigenbaum constant. Explain everything!

**4.** (10 points altogether)
(a) (2 points) Define a **Platonic soild**
(b) (2 points) Let $a$ be the number of edges meeting each vertex, and let $b$ be the number of edges surrounding each face. Express $V$ (the number of vertices) and $F$ (the number of faces) in terms of $E$ (the number of edges), and $a$ and $b$.
(c) (2 points) Find an expressions for $F$, in terms of $a$ and $b$.
(d) (4 points) Obviously both $a$ and $b$ must be at least 3, and $F$ (and hence $V$ and $E$) must be positive. It is easy to see (you don't have to do it) that $a, b$ must be both between 3 and 5, leaving 9 potential scenarios. Find those values of $a$ and $b$ that make sense, and thereby prove that there are exactly 5 Platonic solids. For each of them, find $F$ (the number of faces) and give the name of the corresponding Platonic solid.

**5.** (10 points)
Prove Lagrange's theorem that if $H$ is any subgroup of a group $G$, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

**6.** (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x, y) + iv(x, y)$ where $u(x, y), v(x, y)$ satisfy the above system of two equations?

**7.** (10 points) Who discovered the quaternions? What city did that person live in?

**8.** (10 points) What is Heron's formula, what century did Heron live in?

**9.** (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

**10.** (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

**11.** (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

(b) ( 5 points) State the names of two people who initiated the use of logarithms

**12.** (10 points altogether) (a) (3 points) Define a *Eulerian path* in a graph.

(b) (3 points) State the necessary condition for a graph to have a Eulerian path

(c) (4 points) Prove (or explain in your own words) why the condition in (b) is necessary.