

Homework for Dr. Z.'s MathHistory for Lecture 15

0. Read and understand Chapter VII, sections 8-10 (pp. 186-192), summarize its content in your own words, and your own handwriting, and write it in your HISTORY notebook, [You should have at least the equivalent of two typed pages, but you are welcome to write more]

The other problems should be either hand-written or typed and sent as .pdf file or .txt file (PLEASE no other formats) to DrZlinear@gmail.com by 8:00pm Sunday, Nov. 7, 2021 ,

Subject: hw15

with an attachment: hw15FirstLast.pdf (or hw15FirstLast.txt)

Also in the BODY of the homework, have your name and indicate whether it is OK to post the homework in my web-site.

1. For each of the 6 permutations of $\{1, 2, 3\}$, find the number of inversions.
2. What is the number of inversions of the permutation (written in one-line notation)

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3. Prove that in the fifteen puzzle, if you start with

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & \end{pmatrix}$$

It is **impossible**, by sliding, to get to the position

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 15 & 14 & \end{pmatrix} ,$$

and hence that Sam Lloyd was safe in offering a large prize for its solution.

4. Define a group.
5. Prove that the set of all 2×2 matrices with integer entries, and determinant 1 is a group. **The operation is matrix-multiplication.** Is it a finite group?

In particular, (i) What is the identity element? (ii) how to find the inverse?

(Note: you do not have to prove the associativity, just say that it is known from Linear Algebra.)

6. (a bit of a challenge) Prove that the set of all 2×2 matrices with entries that are in $\{0, 1, 2\}$, with **non-zero determinant**, and where the operation is matrix multiplication done mod 3 is a group. Is it a finite group? How many elements does it have?

(Note: you do not have to prove the associativity, just say that it is known from Linear Algebra)