Attendance Quiz for Review for Exam 1 class

NAME: (print!) ____

E-MAIL ADDRESS: (print!) ____

1. State by do not prove Ore's theorem about Hamiltonian graphs.

Sol. to 1: If a simple graph G with at n edges ($n \geq 3$) has property that for *any* two vertices u and v **not** connected by an edge (in other words $\{u, v\} \notin E$) we have

$$degree(u) + degree(v) \geq n \quad ,$$

then G is Hamiltonian (i.e. has a Hamiltonian cycle)

2. (i) State Dirac's Theorem and (ii) prove that Ore's theorem implies Dirac's theorem.

Sol. to 2(i): If G is a simple graph with n edges ($n \geq 3$) with the property that for *every* vertex u we have

$$degree(u) \geq \frac{n}{2} \quad ,$$

then G is Hamitonian.

Sol. of 2(ii)

We have to show that the *hypotheses* of Dirac imply the hypotheses of Ore, and then invoke Ore.

Let u and v be any two vertices not connected by an edge. Since the hypothesis of Dirac is that *every* vertex had degree $\geq \frac{n}{2}$, we have

$$degree(u) \geq \frac{n}{2} \quad ,$$
$$degree(v) \geq \frac{n}{2} \quad ,$$

Adding these two inequalities we have

$$degree(u) + degree(v) \geq \frac{n}{2} + \frac{n}{2} = n \quad .$$

But this is the hypothesis of Ore, so by Ore, our graph is Hamitonian.

Comment: Many students misunderstoon the quesion and some of the answers were complete gibberish, trying to 'prove' that $degree(u) \geq \frac{n}{2}$. This doesn't make any sense! "$degree(u) \geq \frac{n}{2}$ for very vertex u" is the hypothesis of Diract, not the conclusion! The conclusion is that 'G is Hamitonian'.