

Solutions to Attendance Quiz for Lecture 6

NAME: (print!) Dr. Z.

1. (a) Is K_8 Eulerian? Explain! (b) Is K_9 Eulerian? Explain!

Sol. to 1: (a) All the vertices of K_8 have degree 7, and 7 being odd, of course it is **not** Eulerian.
(b) All the vertices of K_9 have degree 8, 8 being even, it follows from Euler that it is Eulerian.

2. Prove that if G is a graph in which the degree of every vertex is at least 2, then G contains a cycle.

Sol, of 2: Start anywhere you want, say v_1 and keep walking, following any available edge at random.

Since every vertex has degree at least 2 whenever you visit a new (not-visited) vertex, you can continue to another vertex, so your path is

$$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow \dots \quad ,$$

but since the graph is **finite** you can't keep going for ever always to a not-yet-visited vertex, sooner or later you come to a vertex visited before. Suppose it is v_k . Then your itinerary looks like

$$v_1 \rightarrow v_2 \rightarrow \dots v_k \rightarrow v_{k+1} \rightarrow v_{k+2} \dots \rightarrow v_{k+i} \quad ,$$

where $v_{k+i} = v_k$.

Yea! You found a cycle

$$v_k \rightarrow v_{k+1} \rightarrow v_{k+2} \dots \rightarrow v_{k+i-1} \rightarrow v_k \quad .$$

QED.