Solutions to Attendance Quiz for Lecture 5

NAME: (print!) Dr. Z.

1. The *girth* of a graph is the length of its shortest cycle. Write down the girth of

(i) K_{100} (ii) $K_{10,20}$ (iii) Q_{20}

Sol. to 1: (i) 3, (a 2-cycle is not possible, but for any three vertices $1 \leq a, b, c \leq 100$, $a \to b \to c \to a$ is a cycle.

(ii) 4 (The girth of *any* (non-trivial) bipartite graph is 4, a cycle of length 2 is not possible, and it can't have odd cycles, but if a_1, a_2 are any distinct vertices at the top and b_1, b_2 are any distinct vertices at the bottom, $a_1 \to b_1 \to a_2 \to b_2 \to a_1$ is a cycle.

(iii) 4, since Q_k is always bipartite.

2. Write down $\kappa(G)$ and $\lambda(G)$ for

(i) C_{10} (ii) W_{20} (iii) Q_6

Sol. to 2(i)

$\kappa(C_{10}) = 2$: removing one vertex does not disconnect C_{10} but removing two non-adjacent vertices does.

$\lambda(C_{10}) = 2$: removing one edge does not disconnect C_{10} but removing two edges adjacent to the same vertex does.

Sol. to 2(ii)

$\kappa(W_{20}) = 3$: removing one vertex from the perimeter does not disconnect W_{20}, even removing two of them doesn't because of the center, but removing two non-adjacent vertices from the perimeter **and** the center, does disconnet the graph

$\lambda(W_{20}) = 3$: removing one edge does not disconnect W_{20} even removing two edges incident to the same vertex doesn't (because of all the edges between the center to the perimeter, but if removing two edges incident to the same vertex from the perimeter **and** the edge from the center to that vertex does.

Sol. to 3(iii)

$\kappa(Q_6) = 6$. More generally $\kappa(Q_n) = n$ you have to remove n vertices to disconnect it

$\lambda(Q_6) = 6$. More generally $\lambda(Q_n) = n$ you have to remove n edges adjacent to the same vertex to disconnect it.