Attendance Quiz for Lecture 21

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1. There are Four women $1, 2, 3, 4$ and four men a, b, c, d

• Ms. 1 knows Mr a and Mr. b

• Ms. 2 knows Mr b and Mr. c

• Ms. 3 knows Mr a and Mr. d

• Ms. 4 knows Mr c and Mr. d

(i): Check that the conditions of Hall'e Theorem are satisfied, i.e. for each of the 15 non-empty subsets of the st of women, they know collectively at least as many men.

{1} knows {a, b}
{2} knows {b, c}
{3} knows {a, d}
{4} knows {c, d}

{1, 2} know {a, b, c}
{1, 3} know {a, b, d}
{1, 4} know {a, b, c, d}
{2, 3} know {a, b, c, d}
{2, 4} know {b, c, d}
{3, 4} know {a, c, d}

{1, 2, 3} knows {a, b, c, d}
{2, 3, 4} knows {a, b, c, d}
{1, 2, 3, 4} knows {a, b, c, d}

Thus, Hall's condition holds for every nonempty subset S and a perfect matching exists.

(ii)

Currently there are only three married couples:

• Ms. 2 and Mr. b

• Ms. 3 and Mr. a

• Ms. 4 and Mr. c

But poor Ms. 1 she is single, and the only two men she knows (namely Mr. a and Mr. b) are currently married. Use the **alternating path algoritm** to produce four married couples (no credit for other methods).

Let the current matching be M_0 s.t. $M_0 = \{(2, b), (3, a), (4, c)\}$

Start with Ms. 1

$1 \xrightarrow{K} a \xrightarrow{M} 3 \xrightarrow{K} d$

Now, $M_1 = \{(1, a), (2, b), (4, c), (3, d)\}$

Everyone is married!

HQ1) i. $B_1 = \{1, 2\}$

$B_2 = \{2, 3\}$

$B_3 = \{3, 4\}$

$B_4 = \{4, 6\}$

$B_5 = \{1, 5\}$

$B_6 = \{5, 6\}$

ii $B_2 \cup B_4 \cup B_6 = \{2, 3, 4, 5, 6\}$

iii. $q_1 \to B_3$

$q_2 \to B_1$

$q_3 \to B_2$

$q_4 \to B_5$

$q_5 \to B_6$

$q_6 \to B_4$

new matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 6 & 1 & 5 \\ 3 & 1 & 2 & 5 & 6 & 4 \end{pmatrix}$$