Solutions to Attendance Quiz for Lecture 19

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1. (a) State Ramsey's theorem for several colors.

For any given number of colors, c, and any given integers $n_1 \dots n_c$, there is a number, $R(n_1, \dots, n_c)$, such that if the edges of a complete graph of order $R(n_1, \dots, n_c)$ are colored with c different colors, then for some i between 1 and c, it must contain a complete subgraph of order n_i whose edges are all color i.

(b) Using Ramsey's Theorem for two colors, prove it.

We prove a **Lemma**

$$R(n_1, \dots, n_c) \le R(n_1, \dots, n_{c-2}, R(n_{c-1}, n_c)) \quad .$$

Proof of Lemma: Consider a complete graph on $R(n_1, \dots, n_{c-2}, R(n_{c-1}, n_c))$ vertices and color its edges with c colors. Now pretend to be *color-blind* and assume that $c-1$ and c are the same color. By the definition of $R(n_1, \dots, R(n_{c-1}, n_c))$ such a graph must either contain a monochromatic K_{n_i} colored with colored i for some $1 \le i \le c-2$, or a $K_{R(n_{c-1}, n_c)}$ colored in the 'combined-color' $(c-1, c)$. By the two-color Ramsey theorem, it must contain either a monochromatic $K_{n_{c-1}}$ colored with color $c-1$ or a monochromatic K_{n_c} colored with color c. Since by the two-color Ramsey theorem $R(n_{c-1}, n_c)$ is finite the lemma implies, by induction, that for any c, and any positive integers $n_1, \dots, n_c$, $R(n_1, \dots, n_c)$ is finite. QED.