Attendance Quiz for Lecture 13

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1. State and prove Euler's formula relating the number of vertices, edges, and faces, of a planar graph.

Euler's formula is $n-m+f=2$ for n vertices, m edges, f faces

We will prove this by induction:

Base case $(m=0)$: We have a planar graph with 0 edges, and it must have 1 vertex, and 1 face (infinite face) if it is planar.

So $n-m+f=2 \Rightarrow 1-0+1=2 \Rightarrow 2=2 \Rightarrow$ Formula holds for base case.

Inductive Hypothesis: Let G be a graph with n vertices, k edges, and f faces. G is planar, and assume that $n-k+f=2$ is true.

Inductive Step: Remove an edge from G. Then the new graph will have $k'=k-1$ edges, $n'=n$ vertices, and $f'=f-1$ faces, as removing an edge will merge 2 faces into 1, or disconnect it.

So we can see that $n'-k'+f' = n-(k-1)+(f-1)$

$$= n-k+\cancel{1}+f-\cancel{1}$$

$$= n-k+f$$

And according to Inductive Hypothesis this is equal to 2.

So $n'-k'+f'=2$. Hence we proved $n-m+f=2$ for planar graphs by induction $\square$