Attendance Quiz for Lecture 12

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THEOREM 12.1. *$K_{3,3}$ and K_5 are non-planar.*

Remark. We give two proofs of this result. The first one, presented here, is constructive. The second proof, which we give in Section 13, appears as a corollary of Euler's formula.

Proof. Suppose first that $K_{3,3}$ is planar. Since $K_{3,3}$ has a cycle $u \to v \to w \to x \to y \to z \to u$ of length 6, any plane drawing must contain this cycle drawn in the form of a hexagon, as in Fig. 12.2.

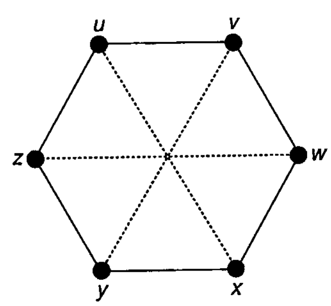

Fig. 12.2

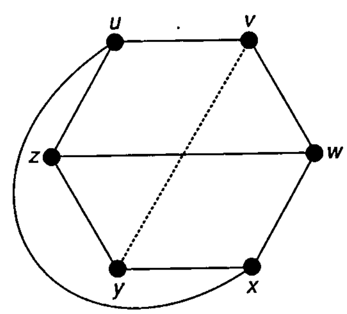

Fig. 12.3

Now the edge wz must lie either wholly inside the hexagon or wholly outside it. We deal with the case in which wz lies inside the hexagon – the other case is similar. Since the edge ux must not cross the edge wz, it must lie outside the hexagon; the situation is now as in Fig. 12.3. It is then impossible to draw the edge vy, as it would cross either ux or wz. This gives the required contradiction.

Now suppose that K_5 is planar. Since K_5 has a cycle $v \to w \to x \to y \to z \to v$ of length 5, any plane drawing must contain this cycle drawn in the form of a pentagon, as in Fig. 12.4.

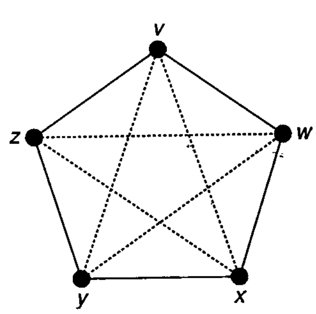

Fig. 12.4

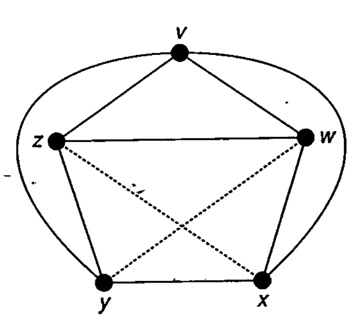

Fig. 12.5

Now the edge wz must lie either wholly inside the pentagon or wholly outside it. We deal with the case in which wz lies inside the pentagon – the other case is similar. Since the edges vx and vy do not cross the edge wz, they must both lie outside the pentagon; the situation is now as in Fig. 12.5. But the edge xz cannot cross the edge vy and so must lie inside the pentagon; similarly the edge wy must lie inside the pentagon, and the edges wy and xz must then cross. This gives the required contradiction. //