NAME: (print!) ________________________________

E-Mail address: ________________

MATH 428 (2), Dr. Z. , Exam 2, Wed., Nov. 26, 2025, 10:20-11:40pm, TILLETT-251

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.
Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 20)

4. (out of 20)

5. (out of 20)

6. (out of 10)

7. (out of 10)

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tot.: (out of 100)

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1. (10 points altogether)
(a) (7 points) Using Euler's formula and the fact that every face must have at least three edges, prove that for a simple planar graph with m edges and n vertices we have

$$m \leq 3n - 6 \quad .$$

(b) (3 points) Using this fact, prove that K_5 is non-planar.

2. (10 points altogether)
(a) (7 points) Using Euler's formula and the fact that every face must have at least three edges, prove that for a simple planar graph that has no triangular faces with m edges and n vertices, we have

$$m \leq 2n - 4 \quad .$$

(b) (3 points) Using this fact, prove that $K_{3,3}$ is non-planar.

3. (20 points) State (3 points) and prove (17 points) the Five-Color Theorem for Planar graphs.

4. (20 points altogether)

(a) (3 points) State Euler's formula relating the number of vertices, edges, and faces of a planar graph.

(b) (17 points) Prove it.

5. (20 points altogether) (a) (5 points) define the **chromatic function** of a simple graph G, $P_G(k)$.

(b) (5 points) *State* the **deletion-contraction** recurrence relation for $P_G(k)$.

(c) (7 points) *Prove* the **deletion-contraction** recurrence relation

(d) (3 points) Use the **deletion-contraction** recurrence relation to prove that $P_G(k)$ is always a polynomial in k.

6. (10 points) In how many ways can you color the vertices of the graph G below with 10 colors?
G is the graph with 4 vertices labeled $\{1, 2, 3, 4\}$ and the set of edges is

$$\{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{1, 4\}, \{1, 3\}\}$$

EXPLAIN everything.

7. (10 points) (a) (3 points) Define the *chromatic index* of a simple graph.

(b) (7 points) Prove König's theorem that states that the chromatic index of a **bipartite** graph equals its largest vertex degree.