NAME: (print!)									
I	-Mail address:								
MAT	MATH 428 (2), Dr. Z., Exam 2, Wed., Nov. 26, 2025, 10:20-11:40pm, TILLETT-251								
No (work	ME YOUR FINAL ANSWER(S) TO EACH PROBLEM alculators! No books! No Notes! To ensure maximum credit, organize your neatly and be sure to show all your work. It write below this line								
1.	(out of 10)								
2.	(out of 10)								
3.	(out of 20)								
4.	(out of 20)								
5.	(out of 20)								
6.	(out of 10)								
7.	(out of 10)								
tot.:	(out of 100)								

This page left blank

1. (10	points	altogether
- · ·	10	POILIO	and Scoring

(a) (7 points) Using Euler's formula and the fact that every face must have at least three edges, prove that for a simple planar graph with m edges and n vertices we have

$$m \leq 3n - 6$$
 .

(b) (3 points) Using this fact, prove that K_5 is non-planar.

2. (10 points altogether)

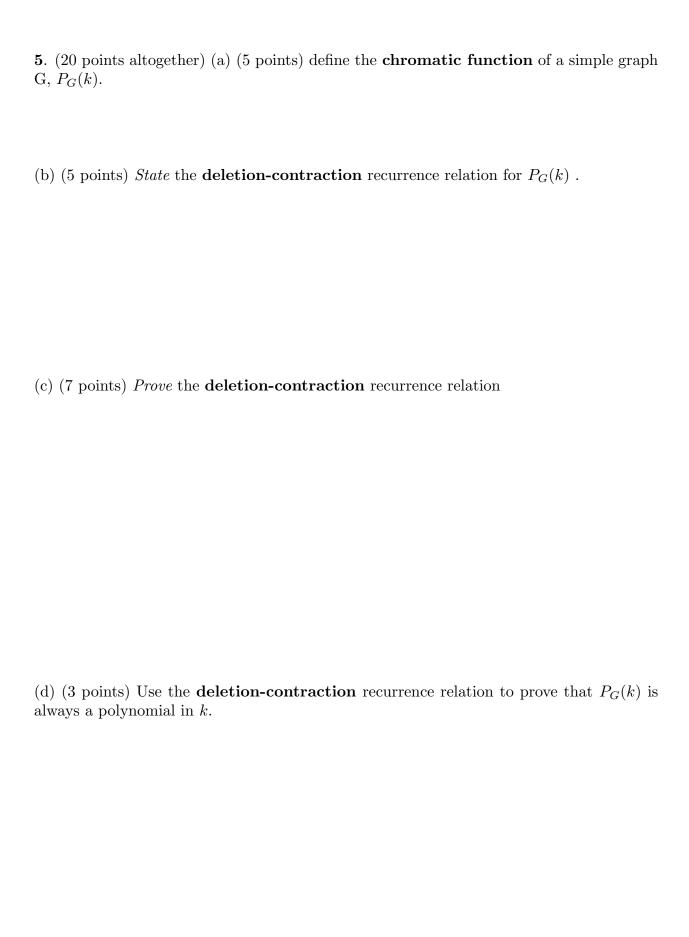
(a) (7 points) Using Euler's formula and the fact that every face must have at least three edges, prove that for a simple planar graph that has no triangular faces with m edges and n vertices, we have

$$m \le 2n - 4$$
 .

(b) (3 points) Using this fact, prove that $K_{3,3}$ is non-planar.

3 . (20 points) State (graphs.	3 points) and	prove (17	points) the	Five-Color	Theorem	for Planar

- 4. (20 points altogether)
- (a) (3 points) State Euler's formula relating the number of vertices, edges, and faces of a planar graph.
- (b) (17 points) Prove it.



6. (10 points) In how many ways can you color the vertices of the graph G below with 10 colors?

G is the graph with 4 vertices labeled $\{1,2,3,4\}$ and the set of edges is

$$\{\{1,2\},\{2,3\},\{3,4\},\{1,4\},\{1,3\}\}$$

EXPLAIN everything.

7. (10 points) (a) (3 points) Define the <i>chromatic index</i> of a simple graph.
(b) (7 points) Prove König's theorem that states that the chromatic index of a bipartite graph equals its largest vertex degree.