

NAME: (print!) _____

E-Mail address: _____

MATH 428 (2), Dr. Z. , Exam 1, Thurs. Oct. 21, 2025, 10:20-11:40pm, TILLET-251

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM

No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.

Do not write below this line

1. (out of 10)

2. (out of 10)

3. (out of 20)

4. (out of 20)

5. (out of 20)

6. (out of 10)

7. (out of 10)

tot.: (out of 100)

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1. (10 points) Prove that $K_{3,3}$ is non-planar.

2. (10 points altogether) (a) (5 points) Draw the Petersen graph

(b) (5 points) Find a Eulerian cycle, or explain why it does not exist.

3. (20 points altogether) (a) (3 points) Define what it means for a simple graph to be *Hamiltonian*

(b) (10 points) State, but do not prove, Ore's theorem about a sufficient condition for a simple graph to be Hamiltonian

(c) (7 points) State Dirac's theorem about a sufficient condition for a simple graph to be Hamiltonian, and show how it follows from Ore's theorem.

4. (20 points altogether)

(a) (10 points) Draw the labeled tree whose Prüfer Code is

1234567

(b) (10 points) Draw the doubly rooted labeled tree whose Joyal Code is

123456789

Indicate the **primary root** and the **secondary root** .

5. (20 points altogether) (a) (10 points) Draw all the *unlabeled trees* with 5 vertices

(b) (10 points) For each of them, state how many ways can you label them. What is the total number?

6. (10 points altotether) (a) (2 points) Draw the graph whose set of vertices is $\{a, b, c, d, e\}$ and whose set of edges is

$$\{\{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, \{a, c\}\} \quad .$$

(b) (8 points) Draw all its spanning trees (if they exist, if not explain why), and if they do not exist draw all the spanning forests.

7. (10 points) Prove that if G is a bipartite graph, then every cycle has even length.