Real Quiz 8

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1. (a) Define a *platonic solid* (b) Prove that there are only five of them, and determine them all.

a) A 3-D figure such that each face is the same polygon of the same size, bounded by the same number of edges, and each vertex has the same degree is a platonic solid.

b) Proof: Let V = # vertices, E = # edges, F = # faces, p = degree of each vertex and q = number of edges bounding each face.

Then $V = \frac{2}{p}E$ and $F = \frac{2}{q}E$. Plugging this into Euler's formula gives $V - E + F = \frac{2}{p}E - E + \frac{2}{q}E = 2$. Then $E(\frac{2}{p} - 1 + \frac{2}{q}) = 2$, $E = \frac{2}{\frac{2}{p}+\frac{2}{q}-1}$. The number of edges cannot be negative so $E \geq 0$, thus $\frac{2}{p} + \frac{2}{q} - 1 > 0$. Then $\frac{2}{p} + \frac{2}{q} > 1$, $\frac{2q+2p}{pq} > 1$, $pq - 2p - 2q < 0$, $(p-2)(q-2) - 4 < 0$, and $(p-2)(q-2) < 4$.

Since the faces of a platonic solid must be at least a triangle, $p \geq 3$ and $q \geq 3$. Thus the only possible values of $(p-2)$ and $(q-2)$ are 3, 2, and 1.

If $p-2=3$ and $q-2=1$, then $p=5$, $q=3$, $V=12$, $E=30$, $F=20$ and the solid is the Icosahedron.

If $p-2=1$ and $q-2=3$, then $p=3$, $q=5$, $V=20$, $E=30$, $F=12$ and the solid is the Dodecahedron.

If $p-2=2$ and $q-2=1$, then $p=4$, $q=3$, $V=6$, $E=12$, $F=8$ and the solid is the Octahedron

If $p-2=1$ and $q-2=2$, then $p=3$, $q=4$, $V=8$, $E=12$, $F=6$ and the solid is the Cube.

If $p-2=1$ and $q-2=1$, then $p=3$, $q=3$, $V=4$, $E=6$, $F=4$ and the solid is the Tetrahedron.

Therefore, these are the only five platonic solids. $\blacksquare$

Real Quiz 8

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1. (a) Define a *platonic solid* (b) Prove that there are only five of them, and determine them all.

a) a platonic solid has all the same degree p, each face is bounded by the same number of edges q, and each adjacent face shares only one edge

b) $E = \frac{pV}{2} \to V = \frac{2E}{p}$

$E = \frac{qF}{2} \to F = \frac{2E}{q}$

$V - E + F = 2 \to \frac{2}{p}E - E + \frac{2}{q}E = 2 \to E\left(\frac{2}{p} - 1 + \frac{2}{q}\right) = 2 \to E = \frac{2}{\frac{2}{p} - 1 + \frac{2}{q}}$

$E > 0$, so $\frac{2}{p} - 1 + \frac{2}{q} > 0$

$\frac{2}{p} + \frac{2}{q} > 1$

$\frac{2q + 2p}{pq} > 1$

$2q + 2p - pq > 0$

$pq - 2q - 2p < 0$

$(p-2)(q-2) - 4 < 0$

$(p-2)(q-2) = \left\{ \begin{matrix} 3 \\ 2 \\ 1 \end{matrix} \right\}.$

There are only 5 solutions to this. The tetrahedron $\{3,3\}$, the cube $\{3,4\}$, the octohedron $\{4,3\}$, the icosahedron $\{3,5\}$, and the dodecahedron $\{5,3\}$.