Solutions Attendance Quiz for Lecture 6

NAME: Dr. Z.

1. (a) Is K_8 Eulerian? Explain! (b) Is K_9 Eulerian? Explain.

Sol. to 1: The degree of every vertex of K_n (the complete graph on n vertices) is $n-1$. Hence if n is odd, all its vertices have even degree and it is Eulerian, and if n is even, all its vertices have odd degree, that implies that not all the vertices have even degree, hence it is not Eulerian.

Hence (a) K_8 is **not** Eulerian (b) K_8 is Eulerian

2. Prove that if G is a graph in which the degree of each vertex is at least 2, then G contains a cycle.

Sol. to 2: If there are multiple edges or loops it is obvious (a loop is a cycle, and if there are two edges between vertex u and v then $u - v - u$ is a cycle).

Start at any vertex, let's call it v, and constuct a walk

$$v \to v_1 \to v_2 \to \quad ,$$

using the following recursive algorithm.

Let v_1 be any vertex adjacent to v (since the degree of v is at least 2, there is such a vertex). Once you have

$v \to v_1 \to v_2 \to \ldots v_{i-1} \to v_i$,

choose v_{i+1} to be any vertex adjacent to v_i **except** v_{i-1}. Such a vertex exists since v_i has at least two neighbors (by our hypothesis).

Since there are **finitely** many vertices, all the v_i's can't be all different, so sooner or later we will encounter a vertex v_k that has been visited before, i.e. $k < k'$ such that $v_k = v_{k'}$.

$$v_k \to v_{k+1} \to \ldots \to v_{k'-1} \to v_{k'} = v_k \quad .$$

This is the desired cycle.