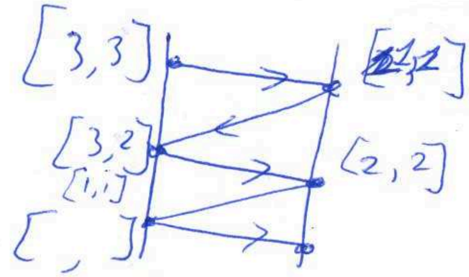

Attendance Quiz for Lecture 4

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1. Using graph theory solve this classical puzzle

Three missionaries and three cannibals must cross a river using a boat which can carry at most two people, under the constraint that, for both banks, if there are missionaries present on the bank, they cannot be outnumbered by cannibals (if they were, the cannibals would eat the missionaries). The boat cannot cross the river by itself with no people on board.

As follows: Construct a **directed** graph with vertices and edges, where a vertex has labels $[m, c]$, with $0 \leq m, c \leq 3$ where m is the number of missionaries in the originating bank, and c is the number of cannibals in the originating bank (hence their numbers in the terminal bank are $3 - m$ and $3 - c$. Of course the number of cannibals on either bank should not exceed the number of missionaries, for obvious reasons.

The edges are

$$[m, c] \to [m-2, c] \quad , \quad [m, c] \to [m, c-2] \quad , \quad [m, c] \to [m-1, c-1] \quad , [m, c] \to [m, c-1] \quad , \quad [m, c] \to [m-1, c]$$

corresponding to all the boat rides from the originating bank to the terminal bank.

(a) List all the vertices?

(b) List all the (directed) edges (for examle $[2, 2] \to [1, 1]$.

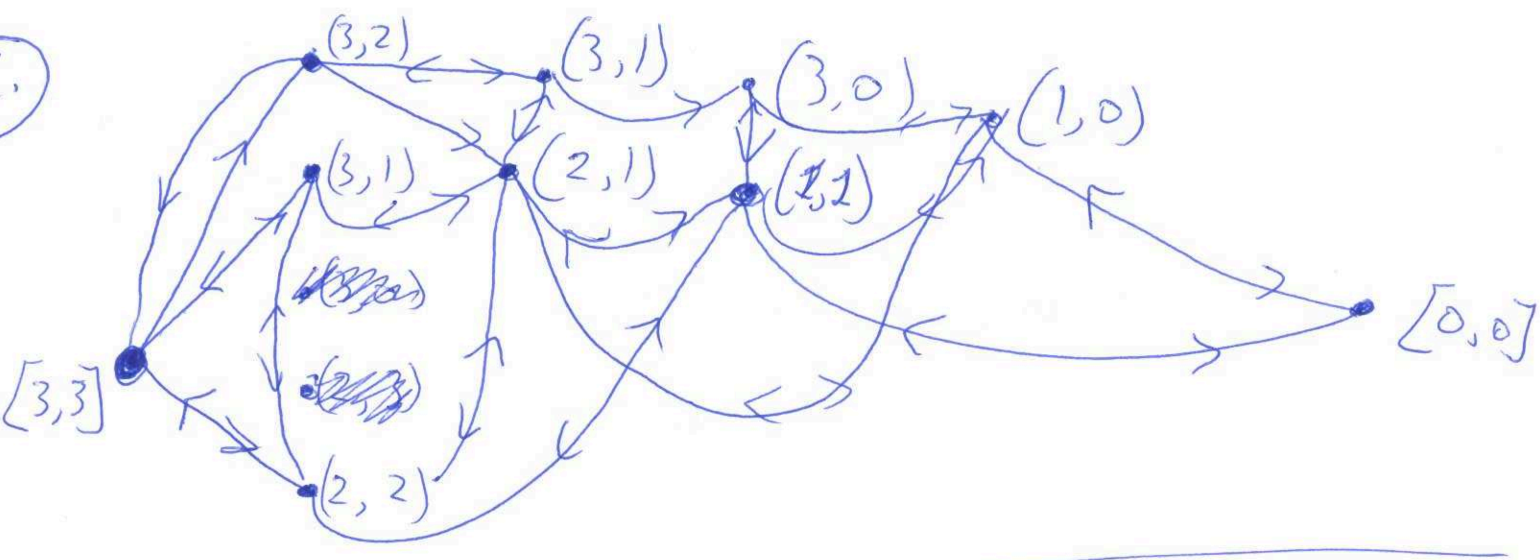
(c) Draw the graph.

(d) Solve the puzzle by finding an **alternating path**: Starting with vertex $[3, 3]$ and ending with vertex $[0, 0]$.

a. [3,3], [3,2], [3,1], [3,0], [2,2], [1,1], [0,0]

b. {[3,3] -> [2,2], [3,3] -> [3,1], [2,2] -> [1,1], [2,2] -> [2,0], [1,1] -> [0,0], [2,0] -> [2,2]}

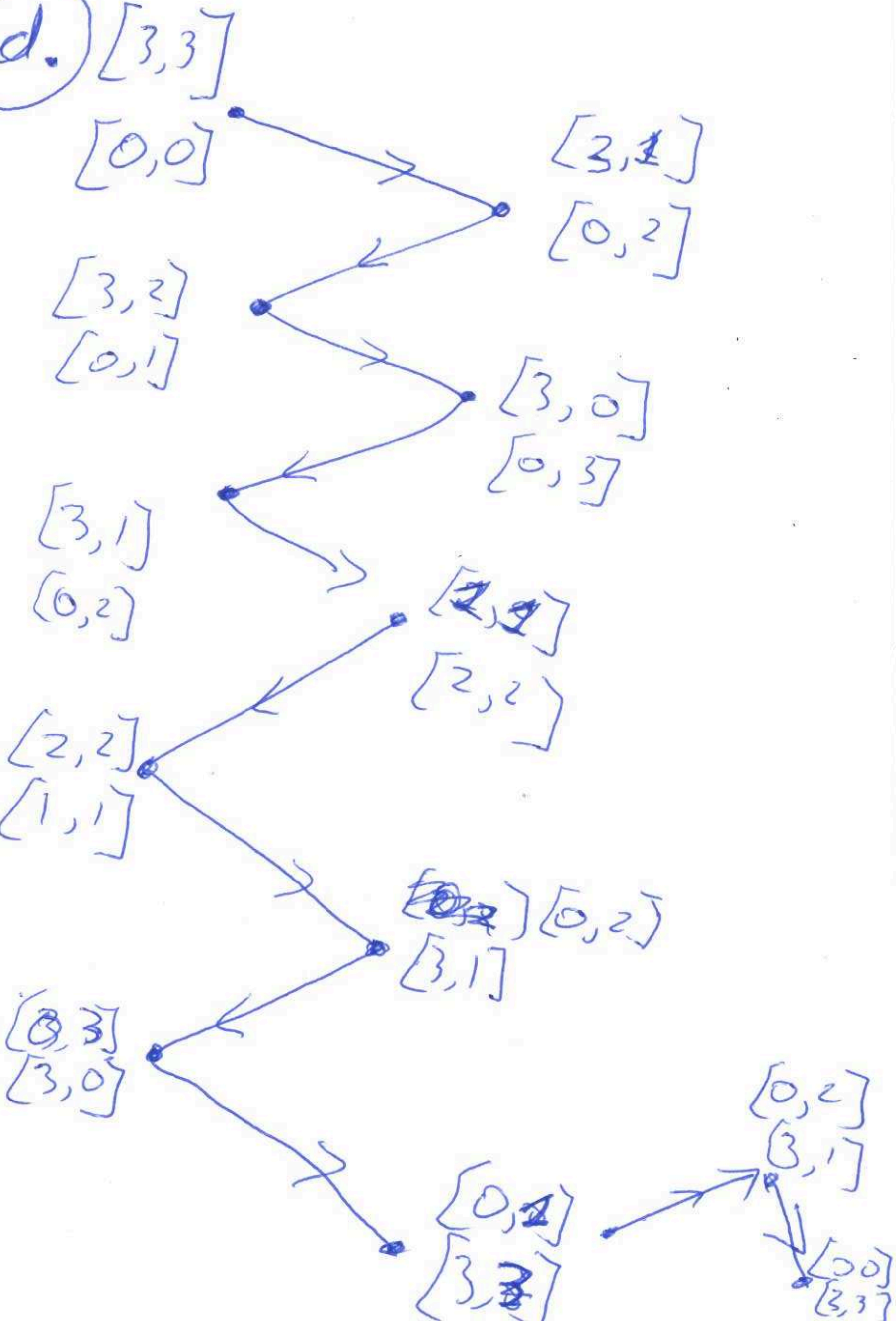
c.



(3,2)
(3,1)
(3,0)
(1,0)
(3,1)
(2,1)
(1,1)
[3,3]
(2,2)
[0,0]

(2,0)
(1,3)
(1,2)
(1,1)
(1,0)
(0,3)
(0,2)
(0,1)

d.



[3,3]
[0,0]
[3,1]
[0,2]
[3,2]
[0,1]
[3,0]
[0,3]
[3,1]
(0,2)
[2,2]
[1,1]
[0,2]
[3,1]
[3,0]
[0,2]
[3,1]
[0,0]
[3,3]