Jason Sanghvi Attendance Quiz

With $m, n > 0$, there are $N$ people s.t. either $m$ people mutually love ~~people~~ eachother or $n$ people mutually hate each other.

$R(m,n) =$ smallest $N$

Let $N = R(m-1,n) + R(m,n-1)$

Prove $R(m,n) \leq N$ by induction

Pick person a, $N-1$ other people

$N-1 = R(m-1,n) + R(m,n-1) - 1$

By pidgeonhole principle either person a

Case 1: loves at least $R(m-1,n)$ people or
Case 2: hates at least $R(m,n-1)$ people

Case 1: ~~Either~~ By induction either
$m-1$ people love each other
$n$ people hate each other

Case 2: By induction either
$m$ people love each other
$n-1$ people hate each other

Factoring in person a, we know at least $m$ people love each other or $n$ people hate each other, no matter which case.

LEONARDO HERNANDEZ lnh306 @ scarlet...

(1) Let $m, n$ be positive integers. Then, there is a $N$ s.t. for every 2-edge coloring of $K_N$ (say red and blue), then there is either a $m$-clique that is all red or an $n$-clique that is all blue.

WOLOG $n \geq m$.

PROOF: BY INDUCTION ON $n$. IT SUFFIC TO PROVE THE FOLLOWING LEMMA:

$R(m, n) \leq R(m-1, n) + R(m, n-1)$.

For $n = 2$: $\left.\begin{array}{l} R(m, 2) = m \\ R(m, 1) = 1 \end{array}\right\}$ by def.

$\Rightarrow$ $m = m-1+1$, SO WE HAVE THE BC.

SUPPOSE WE HAVE THIS FOR $n-1$. THEN BY SYMETRY of $R$ AND BY IH, THE BOUND IS VALID. Let $N = R(m-1, n) + R(m, n-1)$. Let $v \in K_N$. Let $V_R = \{w : wv \text{ is red}\}$

$V_B = \{w : wv \text{ is blue}\}$.

Then $|V_r| + |V_B| = N - 1$

$\Rightarrow |V_r| \geq R(m-1, n)$ (1) or $|V_B| \geq R(m, n-1)$ (2)

by PH principle.

Without loss of generality, suppose (1).

Then $|V_R|$ has a $m-1$ red clique or an $n$-blue clique. If we have $n$-blue clique we are done. Suppose we have an $m-1$ red clique. Then if we include the edges from $V - \{V_R\}$ then we have an $m$-red clique and we are done. The other case is symmetrical so we are done. By def. $R(m,n)$ is the smallest such $N$ with this prop., so $R(m,n) \leq N$ as desired. $\square$