Attendance Quiz for Lecture 17

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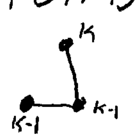
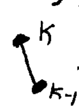
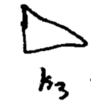
1. What are the chromatic polynomials (in the variable k) of (i) K_n (ii) $\cancel{C_n}$ C_3 (iii) $\cancel{W_n}$ W_4

(i) $P_{K_n}(G) = k(k-1)(k-2)\ldots(k-n+1)$

(ii) $P_{C_3}(G) =$ 

$-$ 

$= k(k-1)^2 - k(k-1)$

(iii) $P_{W_4}(G) =$  $-$  $= ($  $-$ 

$) - P_{C_3}(G)$

$= k \cdot k(k-1)(k-2) - k(k-1)(k-2) - k(k-1)^2 + k(k-1)$

2. Prove that the number of ways to color a simple graph G with k colors is always a **polynomial** in k, that of course depends on G.

Since $P_G(k) = P_{G-e}(k) - P_{G/e}(k)$

and this is recursive to the null graph, which has k^r where r is the number of vertices, G must be a polynomial of degree r.

because if $e = vw$

Case 1: v & w have ~~different~~ color

so $P_{G-e}(k) = P_G(k)$

Case 2: v & w have same color

so $P_{G/e}(k) = P_{G-e}(k)$

So $P_{G-e}(k) = P_G(k) + P_{G/e}(k)$

So $P_G(k) = P_{G-e}(k) - P_{G/e}(k)$