

Attendance Quiz for Lecture 14

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V E F

1. (a) Define a *platonic solid* (b) Prove that there are only five of them, and determine them all.

a) A platonic solid is a convex polyhedron st.

- ① every face is the same regular polygon.
- ② Two adjacent faces share exactly one edge
- ③ ~~each~~ vertex connects the same # of faces.

b)  $p, q$  are resp. values then  $pV = 2E$   
 $\rightarrow$  # edges adjacent to my vertex  
 $\rightarrow$  # edges adjacent to my side  $qF = 2E$

The solids may be projected into planar graphs st.

$$V - E + F = 2 \Rightarrow E = \frac{2}{\frac{1}{p} + \frac{1}{q} - 1} \quad \text{by substituting the above and algebra}$$

Thus  $\frac{1}{p} + \frac{1}{q} - 1 > 0 \Rightarrow (p-2)(q-2) < 4$  by algebra.

(Thus)  $(p-2)(q-2) = \left\{ \begin{matrix} 2 \\ 1 \end{matrix} \right\}$ . Substituting / solving each

of these equations gets us 5 solutions, each one

corresponding to a unique platonic solid:

$\{3, 3\}$  ,  $\{3, 4\}$  ,  $\{4, 3\}$  ,  $\{3, 5\}$  ,  $\{5, 3\}$ .