Attendance Quiz for Lecture 13

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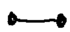
1. State and prove Euler's formula relating the number of vertices, edges, and faces, of a planar graph.

If G is a connected planar graph with v vertices, n edges, and f faces, then $v-e+f=2$.

Proof by induction on number of edges.

$v=1, e=0, f=1$

$v=1, e=1, f=2$ 

$v=2, e=1, f=1$ 

Case 1: Remove 1 edge and still connected. Then $G'=(v',e',f')$. So $v'=v$, $e'=e-1$, $f'=f-1$. Then $v'-e'+f'=v-(e-1)+(f-1)=v-e+f=2$.

Case 2: Remove 1 edge and it disconnects G. So you have 2 components G_1' and G_2' where $G_1'=(v_1',e_1',f_1')$ and $G_2'=(v_2',e_2',f_2')$. Then $v=v_1'+v_2'$, $e=e_1'+e_2'-1$, $f=f_1'+f_2'-1$. Then $v-e+f=(v_1'+v_2')-(e_1'+e_2'-1)+(f_1'+f_2'-1)=2-2+2=2$.