NAME: (print!) ____

E-Mail address: ____________________

MATH 428 (2), Dr. Z. , Exam 2, Thurs., Nov. 30, 2023, 12:10-1:20pm, TILLET-105

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.
Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 20)

4. (out of 20)

5. (out of 20)

6. (out of 10)

7. (out of 10)

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tot.: (out of 100)

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1. (10 points altogether)
(a) (7 points) Use Euler's formula and the fact that each face is bounded by at least three edges, to find an upper bound for the number of edges that a simple planar graph with n vertices can have.

(b) (3 points) Using this fact, prove that K_5 is non-planar.

2. (10 points altogether)
(a) (7 points) Use Euler's formula and the fact that each face is bounded by at least three edges, to find an upper bound for the number of edges that a simple planar graph that has **no triangular faces**, with n vertices can have.

(b) (3 points) Using this fact, prove that $K_{3,3}$ is non-planar.

3. (20 points) Prove the Five-Color Theorem for Planar graphs.

4. (20 points altogether)
(a) (3 points) State Euler's formula relating the number of vertices, edges, and faces of a planar graph.

(b) (17 points) Prove it.

5. (20 points altogether) (a) (5 points) define the **chromatic function** of a simple graph G, $P_G(k)$.

(b) (5 points) (5 points) *State* the **deletion-contraction** recurrence relation for $P_G(k)$

(c) (7 points) *Prove* the **deletion-contraction** recurrence relation

(d) (3 points) Use the **deletion-contraction** recurrence relation to prove that $P_G(k)$ is always a polynomial in k.

6. (10 points) In how many ways can you color the vertices of the graph G below with 10 colors?
G is the graph with 4 vertices labeled $\{1, 2, 3, 4\}$ and the set of edges is

$$\{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{1, 4\}, \{1, 3\}\}$$

EXPLAIN everything.

7. (10 points) (a) (3 points) Define the *chromatic index* of a simple graph.

(b) (7 points) Prove König's theorem that states that the chromatic index of a **bipartite** graph equals its largest vertex degree.