NAME: (print!) ____

E-Mail address: ____________________

MATH 428 (2), Dr. Z. , Final Exam, Thurs., Dec. 21, 2023, 12:00-3:00pm, TILLET-105

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.
Do not write below this line

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1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)
11. (out of 10)
12. (out of 10)
13. (out of 10)
14. (out of 10)
15. (out of 10)
16. (out of 10)
17. (out of 10)
18. (out of 10)
19. (out of 10)
20. (out of 10)

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tot.: (out of 200)

1. (10 points, 2 each) Which of the following graphs are planar? Explain!
(a) $K_{2,4}$ (b) K_3 (c) K_4 (d) K_6 (e) C_8

2. (10 points) If it is your turn to move in a 3-pile Nim, with the the first, second, and third piles having 6, 11, and 15 pennies respectively, what would you do if you want to guarantee to win.

3. (10 points) In the course of using the "keep finding an augmented path until none are left" algorithm to solve the maximum flow problem for a certain network, the following augmented path was found, from the source, vertex 1, to the sink, vertex 9, where the current flow along an edge is indicated in *italics* and its capacity is indicated in **boldface**

$$1 \quad \overset{\mathit{3},\mathbf{7}}{\longrightarrow} \quad 3 \quad \overset{\mathit{5},\mathbf{11}}{\longrightarrow} \quad 2 \quad \overset{\mathit{5},\mathbf{5}}{\longleftarrow} \quad 7 \quad \overset{\mathit{3},\mathbf{5}}{\longleftarrow} \quad 5 \quad \overset{\mathit{6},\mathbf{17}}{\longrightarrow} \quad 9 \quad .$$

(a) (2 points) Explain why this is a legal augmenting path. (b) (6 points) Find the new flow along each edge. (c) (2 points) By how much did it increase?

4. (10 points) (a) (2 points) Give an example of a 3×5 Latin rectangle, using the symbols $1, 2, 3, 4, 5$.
(b) (3 points) Calling the columns the 'girls' (Ms. 1, ..., Ms. 5), and the entries the 'boys' (Mr. 1, ..., Mr. 5), formulate, in the *matrinomy* language, the problem of continuing the 3×5 Latin rectangle into a 4×5 Latin rectangle, by adding a new row on top.
(c) (5 points) Find a matching, and use it to creatie a 4×5 Latin rectangle whose bottom three rows are the same as in (a).

5. (10 points, 2 each) Which of the following graphs have a Eulerian cycle? Eulerian path? Explain
(a) K_{10} (b) K_{101} (c) P_{100} (the path with 100 vertices (d) C_{1001} (the cycle with 1001 vertices) (e) The Petersen graph

6. (10 points) (a) (5 points) State Hall's theorem about Stable Matchings, in the language of matrimony (with girls and boys some of whom know each other) (b) (5 points) State Hall's theorem about Stable Matchings, in the language of transversals where you are given n subsets of $\{1, \ldots, n\}$, $S_1, \ldots S_n$.

7. (10 points) Prove Hall's theorem. You can use any (correct) proof that you know.

8. (10 points) A certain simple graph with 101 vertices has the property that it is regular of degree 51 (i.e. every vertex has exactly 51 neighbors). Can you conclude that it has a Hamiltonian cycle? If yes, explain what theorem you are using. If no, also explain why not.

9. (10 points) In a party of 20 people, any of the 190 pairs of people either love each other or hate each other (they are never indifferent). Are you guaranteed that you can either find 4 people who all love each other (i.e. all the 6 relationships between them are of love) or 4 people who all hate each other (i.e. all the 6 relationships between them are of hate). Explain!

10. (10 points). Construct a party with 5 guests, such that you can **neither** find three people who love each other **nor** three people who hate each other. Calling these people $1, 2, 3, 4, 5$, indicate for each of the ten pairs of guests whether they love each other or hate each other.

11. (10 points). Find the labeled tree, on 9 vertices, whose Prüfer code is 8132782 .

12. (10 points). Find the doubly-rooted labeled tree, on 9 vertices whose Joyal code is 124319238, indicate the primary root and the secondary root.

13. (10 points) List all the unlabeled trees with 5 vertices. For each of them describe the number of ways of labeling them. Add them up. Is the answer consistent with Cayley's theorem?

14. (10 points) What is the chromatic index of (a) K_{1001}? (b) K_{5000}? Explain.

15. (10 points, 2 each) Does there exist a simple graph with largest vertex degree 100 whose chromatic index

(a) 90 (b) 100 (c) 101 (d) 103 (e) 122?

For each of them either give an example, or state which theorem you are using to explain why such graphs do not exist.

16. (10 points) Does there exist a bipartite graph with largest vertex degree 1000 and chromatic index 1001?. State the name of the theorem that you are invoking.

17. (10 points) A simple planar graph with n vertices only has hexagonal faces. How many edges does it have? Can n be odd?

18. (10 points) Prove that every simple planar graph contains a vertex of degree at most 5.

19. (10 points) What is the chromatic polynomial of (i) K_n (the complete graph on n vertices) (ii) N_n (the null graph in n vertices)

20. (10 points) In how many ways can you color P_{10} (the path with 10 vertices) with 10 colors?