There is a series of Erdos-Renyi papers on the subject, but the most concise formulation I found was in the 1961 paper (attached, page 3 bottom). Here the authors state:

\[ N(n) \sim cn, \quad cn < 1/2 \quad \Rightarrow \quad |S| \approx \frac{1}{\alpha} \left( \log(n) - \frac{5}{2} \log(\log(n)) \right) \]  \hspace{1cm} (1)

\[ N(n) \sim n/2, (i.e., c = 1/2) \quad \Rightarrow \quad |S| \approx n^{2/3} \]  \hspace{1cm} (2)

\[ N(n) \sim cn, \quad cn > 1/2 \quad \Rightarrow \quad |S| \approx G(c)n \]  \hspace{1cm} (3)

Where \( N(n) \) is the (expected?) number of edges of a random graph with \( n \) vertices, \( |S| \) is the size of the largest connected component, \( c \) is some constant, \( 0 \leq c \leq 1 \), \( \alpha = 2c - 1 - \log(2c) \), and \( G(c) \) is the function

\[ G(c) = 1 - \frac{1}{2c} \sum_{k=1}^{\infty} \frac{k^{k-1}}{k!} (2ce^{-2c})^k \]

To translate from \( c \) to \( \epsilon \), I think we can take \( N(n) = \binom{n}{2}p \), where \( p \) is the probability of an edge between two vertices. Letting \( p = pc(1 + \epsilon) \), with \( pc = 1/(n-1) \), we then get \( N(n) = n \cdot (1 + \epsilon)/2 \). Or, in other words, \( c = (1 + \epsilon)/2 \).

Then the above set of equations becomes

\[ \epsilon < 0 \quad \Rightarrow \quad |C| \approx \frac{\log(n) - \frac{5}{2} \log(\log(n))}{\epsilon - \log(1 + \epsilon)} \]  \hspace{1cm} (4)

\[ \epsilon = 0 \quad \Rightarrow \quad |C| \approx n^{2/3} \]  \hspace{1cm} (5)

\[ \epsilon > 0 \quad \Rightarrow \quad |C| \approx G(\epsilon)n \]  \hspace{1cm} (6)

Of course with \( G(\epsilon) \) the appropriate substitution of \( c \) to \( \epsilon \).

I did a few trials using these approximations and the results seem much better (ratios approx 0.7 to 1.2), but I’m not sure how much better, in the long run, because I was only able to do some trials with relatively small numbers.

For calculating \( G \), I evaluated in maple with \( k = 1..1000 \) since I couldn’t find a better way to do it.