1. seq(nops(States(i)), i=1..10) is [1, 3, 9, 25, 67, 175, 449, 1137, 2851, 7095] = A106514, "Expansion of (1-x)/((1-2x)(1-2x-x^2))."

   guessgf([1, 3, 9, 25, 67, 175, 449, 1137, 2851, 7095], x) also yields

$$\left[-\frac{-1+x}{3x^2+1-4x+2x^3}, ogf\right].$$

2. First, we notice the following recursion:

$$S_{n+2} = 3S_{n+1} - 2\left(\sum_{i=1}^{n-1} S_i\right).$$

   Increment by 1:

$$S_{n+3} = 3S_{n+2} - 2\left(\sum_{i=1}^{n} S_i\right).$$

   Subtracting the former from the latter, the summation vanishes:

$$S_{n+3} - S_{n+2} = 3S_{n+2} - 2S_n - 3S_{n+1},$$

   or

$$S_{n+3} - 4S_{n+2} + 3S_{n+1} + 2S_n = 0.$$

   This easily gives the desired generating function, but since the coefficients are integers, we can actually cheat and find an explicit formula for S_n. The solutions for

$$x^3 - 4x^2 + 3x + 2 = 0$$

   are

$$x = (2, 1+\sqrt{2}, 1-\sqrt{2}),$$

   hence the general form of the solution is

$$S_n = C_1 2^n + C_2 (1+\sqrt{2})^n + C_3 (1-\sqrt{2})^n,$$

   and substituting S_0=1, S_1=3, and S_2=9, we get

$$C_1 = \text{-}2, C_2 = \frac{3}{2}+\sqrt{2}, C_3 = \frac{3}{2}-\sqrt{2}.$$

   The last term is a rounding term, so we finally have

$$S_n = -2^{(n+1)} + \mathrm{round}\left(\left(\frac{3}{2}+\sqrt{2}\right)(1+\sqrt{2})^n\right).$$